Reliability of settlement prediction based on monitoring

Fiabilité de la prédiction de tassement basée sur l'auscultation

Ed Calle, Hans Sellmeijer & Marcel Visschedijk
GeoDelft, the Netherlands

ABSTRACT
This paper presents an alternative method to predict expected mean values and standard deviations of embankment settlement, as function of time. The method is based on both prior assumptions regarding expected means and standard deviations of settlement parameters and computation model uncertainty, as well as actually observed settlement behavior, e.g. during the construction stage, applying a Bayesian updating concept. Potential applications concern e.g. design of a monitoring strategy and philosophy. The method is still under development, however the first results, demonstrated in this paper, look promising.

RÉSUMÉ
Cet article présente une méthode alternative pour prédire les valeurs moyennes et les écarts-types prévus pour le tassement d'un remblai, en fonction du temps. La méthode est basée sur deux hypothèses préalables concernant les paramètres de tassement moyen et d'écart-type et l'incertitude du modèle de calcul, ainsi que le comportement de tassement effectivement observé, par exemple en appliquant le concept bayesien en cours de construction. Les applications potentielles concernent par exemple la conception de la stratégie et de la philosophie de l'auscultation. Bien que la méthode soit encore en cours de développement, les premiers résultats présentés dans cet article apparaissent prometteurs.

1 INTRODUCTION

Accurate prediction of long term settlements of embankments, raised for road or railway construction or river flood defenses is of significant importance for life cycle cost analysis and decision making about the design. Yet long term settlement prediction usually suffers from substantial uncertainty, due to limited information on key parameters, variability of these parameters and uncertainty involved in the prediction model. It is widely recognized that monitoring of settlement behavior during embankment construction provides additional information which may be helpful to adapt long term settlement prediction, possibly necessitating revision of road or railway foundation design or, in the case of flood levees, adaptation of additional crest height, needed for compensation of settlement at the long term.

Methods used for updating of key parameters for settlement prediction are frequently based on back analysis or inverse parameter modeling, based on observed settlement behavior during embankment construction. Application of a Bayesian framework for this purpose yields the so-called weighted least squares method. This method minimizes not only the residuals between measurements and predictions, but also the residuals between the initial and updated estimate of the mean value of the key parameters. The resulting updated key parameters can be applied to determine a best guess prediction of long term settlements.

Least squares or weighted least squares approaches allow for estimation of prediction uncertainties, based on analysis of residuals between observations and updated predictions. It is intuitively felt, however, that observations of only a small part of the settlement process provide poor information about parameters which essentially govern only the long term part of the process. Relying on residuals of the fitting procedure only, may therefore lead to significant underestimation of long term uncertainty margins.

This paper describes an alternative approach for determination of uncertainty margins using short time observations, by a Bayesian type of updating of long term predictions. Key feature of this approach is that uncertainty estimates in the long term predictions are affected by observations, but only as far as the observations are meaningful for the parameter in question. It is believed that this approach yields more reliable and likely less optimistic, re-estimates of uncertainties of long term settlement predictions.

The method is still under development and as such this paper reports ongoing research. However from the results obtained until yet, the method seems promising.

In the sequel, attention will be first focused at the mathematical background of the method, followed by an examples of computation results. The mathematical background has been set up in a general way, because it is believed that the method may be useful in a wider field of process monitoring than settlements only.

2 THE BAYESIAN FRAMEWORK

A process consists of a set of mutually related characteristics. For instance, the behavior of an embankment is characterized by settlement, stress, pore pressure, etc. These characteristics are piled into a process vector, indicated by **z**:

$$\mathbf{z}(\mathbf{a};t) \;=\; \left(z_1(\mathbf{a};t), \ldots, z_m(\mathbf{a};t), \ldots, z_M(\mathbf{a};t)\right)^{\mathrm{T}} \tag{1}$$

where t is the elapsed time and **a** is a vector of parameters involved in the computation model, including computation model uncertainty parameters.

The elements of the parameter vector **a** may or may not be correlated. Therefore, a covariance matrix is assumed:

$$\mathbf{C}_a \;=\; \begin{bmatrix} \sigma^2(a_1) & \ldots & \mathrm{cov}(a_1,a_n) & \ldots & \mathrm{cov}(a_1,a_N) \\ \ldots & & \ldots & & \ldots \\ \mathrm{cov}(a_n,a_1) & \ldots & \sigma^2(a_n) & \ldots & \mathrm{cov}(a_n,a_N) \\ \ldots & & \ldots & & \ldots \\ \mathrm{cov}(a_N,a_1) & \ldots & \mathrm{cov}(a_N,a_n) & \ldots & \sigma^2(a_N) \end{bmatrix} \tag{2}$$

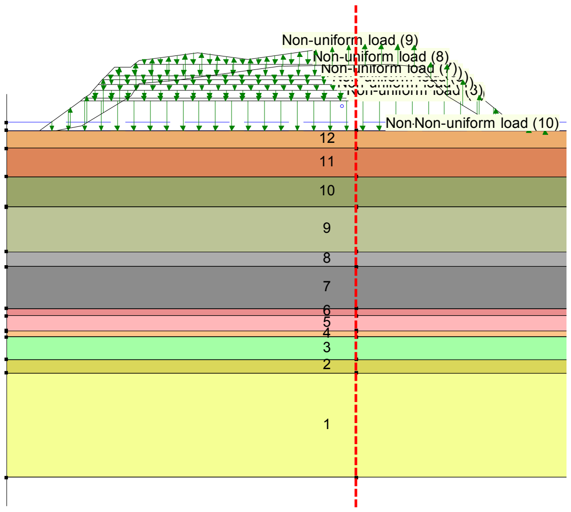

Figure 1: Embankment construction stages and subsoil profile

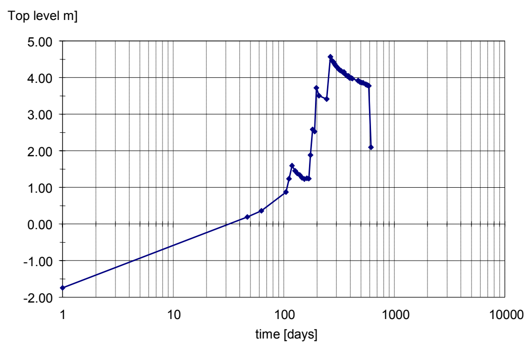

Figure 2: Measured crest height of the embankment vs time

Besides uncertainty of the parameters, the vectors of measurements $\mathbf{z}_{\mathrm{m}}$ and predictions $\mathbf{z}_{\mathrm{p}}(\mathbf{a})$ will usually show a distinction. This is due to measurement errors, reflected by the vector $\mathbf{s}$.

$$\mathbf{z}_{\mathrm{m}} = \mathbf{z}_{\mathrm{p}}(\mathbf{a};\mathbf{t}_{\mathrm{m}}) + \mathbf{s} \tag{3}$$

where $\mathbf{t}_{\mathrm{m}}$ is the vector with M measurement points of time and $\mathbf{s}$ is a vector of zero mean normally distributed random variables. The covariance of the parameters is therefore supplemented with covariances of the measurement errors.

$$\mathbf{C}_{\mathrm{s}} = \begin{bmatrix} \sigma^2(s_1) & \dots & 0 & \dots & 0 \\ \dots & & \dots & & \dots \\ 0 & \dots & \sigma^2(s_{\mathrm{m}}) & \dots & 0 \\ \dots & & \dots & & \dots \\ 0 & \dots & 0 & \dots & \sigma^2(s_{\mathrm{M}}) \end{bmatrix} \tag{4}$$

3 UPDATING OF EXPECTED MEAN VALUES

Linearization of equation (3) yields:

$$\mathbf{z}_{\mathrm{m}} = \mathbf{J}\mathbf{a} + \mathbf{s}, \quad \mathbf{J} = \nabla_{\mathrm{a}} \mathbf{z}_{\mathrm{p}}(\mathbf{a}) \tag{5}$$

The Jacobian type matrix $\mathbf{J}$ consists of derivatives of the prediction $\mathbf{z}_{\mathrm{p}}$ with respect to the parameters $\mathbf{a}$. Equation (6) gives the weighted least squares expression that has to be minimized. Equation (7) gives the direct parameter update in case of a linear relationship between prediction and parameters. Equation (8) gives the iterative parameter update in case of a nonlinear relationship.

$$S = (\mathbf{z}_{\mathrm{m}} - \mathbf{J}\mathbf{a})^T \mathbf{C}_{\mathrm{s}}^{-1} (\mathbf{z}_{\mathrm{m}} - \mathbf{J}\mathbf{a}) + (\mathbf{a} - \mathbf{a}_0)^T \mathbf{C}_a^{-1} (\mathbf{a} - \mathbf{a}_0) \tag{6}$$

$$\mathbf{a}^{(1)} = \left(\mathbf{J}^T \mathbf{C}_s^{-1} \mathbf{J} + \mathbf{C}_{\mathrm{a}}^{-1}\right)^{-1} \left(\mathbf{J}^T \mathbf{C}_{\mathrm{s}}^{-1} \mathbf{z}_{\mathrm{p}} + \mathbf{C}_{\mathrm{a}}^{-1} \mathbf{a}^{(0)}\right) \tag{7}$$

$$\mathbf{a}^{(i+1)} = \mathbf{a}^{(i)} + \left(\mathbf{J}^{(i)T} \mathbf{C}_{\mathrm{s}}^{-1} \mathbf{J}^{(i)} + \mathbf{C}_{\mathrm{a}}^{-1}\right)^{-1} \left(\mathbf{J}^{(i)T} \mathbf{C}_{\mathrm{s}}^{-1} \left(\mathbf{z}_{\mathrm{m}} - \mathbf{z}_{\mathrm{p}}(\mathbf{a}^{(i)})\right) + \mathbf{C}_{\mathrm{a}}^{-1} \left(\mathbf{a}^{(0)} - \mathbf{a}^{(i)}\right)\right) \tag{8}$$

The addition of the weighted residuals between initial and updated parameter values increases robustness of the least squares method. The robustness is improved further by combination with the Levenberg-Marquardt method (Levenberg, 1944), (Marquardt, 1963).

4 UPDATING OF PREDICTION COVARIANCES

The prior prediction of the covariance of settlement vector $\mathbf{z}$ is given by equation (9).

$$\mathbf{C}_{\mathrm{z}} = \mathbf{J}\,\mathbf{C}_{\mathrm{a}}\,\mathbf{J}^T \tag{9}$$

Now assume that $\mathbf{J}$ is determined at the updated mean values of the parameters, and that it consists of a first part for the measurements point of time t_{m} and an additional part with the prediction points of time t_{p}. Then equation (9) may be rewritten into equation (10), with an added contribution of $\mathbf{C}_s$ for the measurement time points. The posterior covariance prediction for all prediction points of time is finally given by equation (11).

$$\mathbf{C}_{\mathbf{z}} = \begin{bmatrix} \mathbf{J}_{\mathbf{m}}\,\mathbf{C}_{\mathbf{a}}\,\mathbf{J}_{\mathbf{m}}^{\mathbf{T}} + \mathbf{C}_{\mathbf{s}} & \mathbf{J}_{\mathbf{m}}\,\mathbf{C}_{\mathbf{a}}\,\mathbf{J}_{\mathbf{p}}^{\mathbf{T}} \\ \mathbf{J}_{\mathbf{p}}\,\mathbf{C}_{\mathbf{a}}\,\mathbf{J}_{\mathbf{m}}^{\mathbf{T}} & \mathbf{J}_{\mathbf{p}}\,\mathbf{C}_{\mathbf{a}}\,\mathbf{J}_{\mathbf{p}}^{\mathbf{T}} \end{bmatrix} \tag{10}$$

$$\mathbf{C}_{\mathbf{z_p}|\mathbf{z_m}} = \mathbf{J}_{\mathbf{p}}\,\mathbf{C}_{\mathbf{a}}\,\mathbf{J}_{\mathbf{p}}^{\mathbf{T}} - \mathbf{J}_{\mathbf{m}}\,\mathbf{C}_{\mathbf{a}}\,\mathbf{J}_{\mathbf{p}}^{\mathbf{T}} \left(\mathbf{J}_{\mathbf{m}}\,\mathbf{C}_{\mathbf{a}}\,\mathbf{J}_{\mathbf{m}}^{\mathbf{T}} + \mathbf{C}_{\mathbf{s}}\right)^{-1} \mathbf{J}_{\mathbf{p}}\,\mathbf{C}_{\mathbf{a}}\,\mathbf{J}_{\mathbf{m}}^{\mathbf{T}} \tag{11}$$

Note that the measurements itself are not explicitly involved in the expression. However, the goodness of fit (Larsen, 1986) can be used to determine the posterior values of the variance in the covariance matrix $\mathbf{C}_{\mathrm{s}}$.

$$\sigma^2(s) = \frac{\mathbf{s}^T \mathbf{s}}{M - N} \tag{12}$$

The diagonal terms of the resulting covariance (11) will be applied to determine the posterior uncertainty margin of the predicted settlements, given the measured settlement data.

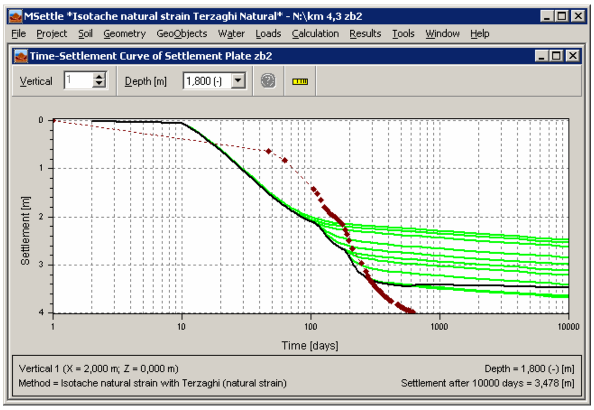

Figure 3: Predicted settlements (expected mean values) before updating based on monitoring data

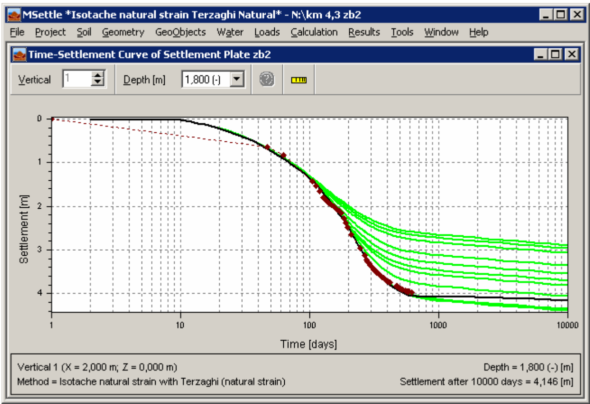

Figure 4: Predicted settlements (expected mean values) after updating based on monitoring data

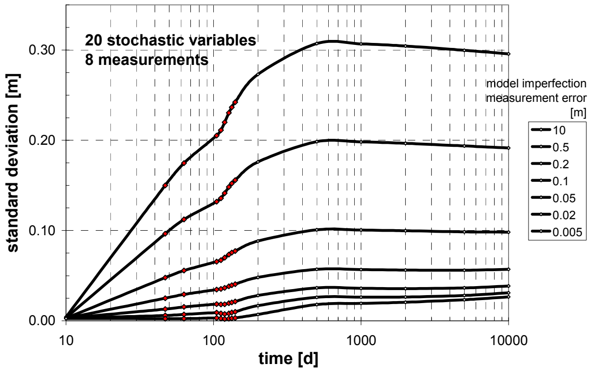

Figure 5: standard deviation of the prediction as function of time for 8 measurement time points

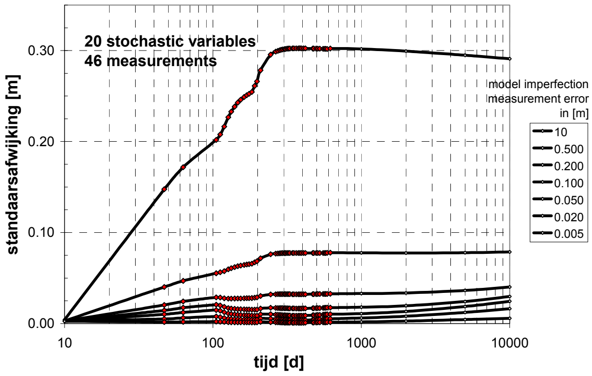

Figure 6: standard deviation of the prediction as function of time for 46 measurement time points

5 EXAMPLE

Embankments are subjected to long term creep behavior of the subsoil. Calculation results in predictions for settlements, pore pressures and effective stresses. Especially, the uncertainty of the final settlements is a point of concern. Thereto, settlements are monitored during and after construction. Sometimes pore pressures are monitored, too. These measurements are applied to reduce the uncertainty of the predictions. In the example it will be shown in how far the uncertainty margins will be reduced due to monitoring of settlements.

The geometry of the example is shown in *figure 1*. The subsoil consists of different soft soil layers, which will settle due to the weight of the embankment raise (*figure 2*). The settlements are determined along a vertical, using the Isotache model (Den Haan, 2000). The special excess pore pressure model includes the influence of vertical drains (Sellmeijer, 2002). Initial estimates of the compression parameters are determined from correlations with the saturated densities. The standard deviations are obtained from a regional collection of soil data. *Figure 3* shows the resulting settlement pattern of the settlements using the initial parameters. The green lines indicate the settlement that would occur without further change of loading. The red dots indicate the measured settlements. *Figure 4* shows the predicted settlements after fitting with the weighted least squares method. The fit was executed with 5 independent fit parameters, being multiplication factors for the parameters of all separate layers. The estimate for the goodness of fit is determined according to equation (12) and equals 0.042. This quantity has up to now been taken to be a measure for assessment of the measurement error, however, this is by no means an obvious choice.

In figure 5 the standard deviation (square root of the variance) is plotted against time, using only the first eight measurement time points. Figure 6 shows the standard deviation using all 46 measurement points of time.

The time points of the measurements are represented by red diamonds. The lines represent different values for the model imperfection or measurement error. These values are indicated at the right hand side. The (conditional) standard deviation is determined, provided there is a certain agreement between prediction and measurement. It makes only sense for points of time beyond the measurements. However, from theoretical point of view, an uncertainty bound for the measurements, based on the measurements, is interesting.

Figures 5 and 6 demonstrate clearly the influence the magnitude of the assumed measurement errors and the influence of the number of observed measuring data. Large measurement errors diminish the influence of observed data, i.e. predictions of settlements can hardly be improved by monitoring of the settlement process. For small measurement errors it can be seen from figures 5 and 6 that variances of uncertainties of predictions of long term settlement drastically decrease as the number of observations increase. In this example no additional computation model imperfection has been taken into account.

6 CONCLUDING REMARKS

The Bayesian approach to determine variances of uncertainty margins of settlement predictions, accounting for monitoring data, is complementary to the usually applied weighted least squares approach for (inverse) parameter assessment, based on monitoring data. Uncertainty of long term settlement prediction, and how it is reduced by monitoring, is a key factor in the decision about whether or not to install monitoring equipment and, if, for how long the settlement process should be monitored. The example shown in this paper demonstrates clearly the influence of the number of measuring data in time on the uncertainty margins of long term predictions of settlements. It appears that the assumptions made about model imperfections and measuring errors have a significant influence. It is not obvious yet how to objectively assess these parameter; this needs further research.

ACKNOWLEDGEMENT

This study is part of the project GeoSafe (development of risk and reliability based tools for geotechnical design), which is financially supported by the Dutch ministry of economic affairs via Senter, grant TSGE3017. The authors thank Dr. Virginie Trompille (GeoDelft) for cooperation in preparing this paper.

REFERENCES

Den Haan, E.J. and Sellmeijer, J.B., 2000. Calculation of soft ground settlement with an isotache model. *"Soft Ground Technology", ASCE Geotech. Spec*. Publ. Nr 112, pp 94-104, 2000

Larsen, R. J., and M. L. Marx, 1986. An Introduction to Mathematical Statistics and Its Applications, Prentice-Hall Inc., Englewood Cliffs, New Jersey.

Levenberg, K., 1944. A method for the solution of certain nonlinear problems in least squares. *Quart. Appl. Math., 2*, 164–168, 1944.

Marquardt, D.W., 1963. An algorithm for least squares estimation of nonlinear parameters. *SIAM J. Appl. Math., 11*, 431–441.

Sellmeijer, J.B., 2002. Vertical Drains simulated as Leakage. In: Learned and Applied Soil Mechanics out of Delft. (Eds. F.B.J. Barends & P.M.P.C. Steijger) pp75-80. Balkema Publishers, ISBN90 5808 357 3.