Investigating the Streaming Potential Phenomenon Using Electric Measurements and Numerical Modelling with Special Reference to Seepage Monitoring in Embankment Dams



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Abstract

Streaming potentials are one of four possible electrokinetic phenomena that relate electric currents with the relative movement of solid and liquid phases in contact with each other. The streaming potential phenomenon describes the occurrence of electric potential differences when a liquid moves with respect to a solid that it is contact with. In geophysical terms, electric potential differences are developed in the ground wherever groundwater movements occur in deposits of soil or porous rock.

The naturally occurring electric potential differences caused by streaming potentials as well as a number of other possible electrochemical sources are also known as selfpotentials due to the absence of any artificially injected electric current. The selfpotential (SP) method involves measuring the electric potential differences between points on the ground with specialised electrodes. With knowledge of the streaming potential phenomenon as well as the other possible sources of self-potentials, the measurements can be interpreted to provide valuable information concerning, for example, groundwater flow patterns or mineral prospecting.

The streaming potential phenomenon is the electrochemical mechanism of interest for SP investigations of groundwater flow and is thus explained in detail for this thesis. The main difficulty in interpreting self-potential measurements lies in the complexity of the electric current sources and the flows they generate. The current sources are a function of the fluid flow and a cross-coupling conductivity that relates the hydraulic and the electric potential differences. The current flows are a complex function of the electric conductivity or resistivity distribution in the ground. For this reason knowledge of subsurface electrical properties is a valuable tool for interpreting SP measurements and thus a discussion of the earth resistivity method is also presented in this work.

In order to provide a better understanding of the sources of SP anomalies, a threedimensional finite element computer program was developed for this thesis to numerically model the streaming potential phenomenon. The program can in fact calculate and display the primary and secondary potential distributions for any two coupled flows in a three dimensional domain. For streaming potentials, the primary flow is hydraulic and the secondary flow is electrical.

The program operates in three separate stages. The program first determines the hydraulic potential distribution in the ground based on hydraulic conductivity values and the hydraulic driving forces, such as the pressure drop through an embankment dam. The program then calculates the geometry and magnitude of the electric current sources based on the fluid flow and cross-coupling conductivity values. Finally the electric potential distribution is solved for using these current sources and the electric conductivity distribution. Additionally, the program can incorporate external current sources, which can be used to simulate resistivity measurements in a model.

The model domain can take any three-dimensional shape and can be divided into elements as desired. The individual elements can be assigned separate hydraulic, electric and cross-coupling conductivity values, creating an inhomogeneous anisotropic domain with three separate conductivity distributions. Four different types of finite element are available to choose from; two- and three-dimensional versions of isoparametric elements with either linear or quadratic interpolating polynomials. The program has been made fully graphical, allowing the user quick and easy access to information at any particular point of the domain.

The hydraulic potential distributions obtained by the program match closely with analytical solutions. The program can also determine the location of the phreatic surface throughout the model domain while calculating the hydraulic potential distribution. The electric potential distributions reflect the calculated conduction current sources as well as variations in the electric and cross-coupling conductivity distributions. The results from simple models containing point pressure sources and sinks matched well with those from the finite difference electric potential program SPPC as well as analytical solutions. Models simulating real earth dam conditions produced potential distributions that showed reasonable agreement with field measurements.

In order to provide a better picture of the streaming potentials in earth dams and the potential of the SP method for dam safety monitoring, SP investigations were performed on a number of embankment dams. Electric resistivity measurements were also performed on some of the dams to compliment the SP data. The resistivity data was found to be of considerable assistance for interpreting the SP measurements as well as for simulating real dam conditions with the modelling program.

Three hydro-electric dams of different size on the Luleå River in northern Sweden were studied together with several dams built by mining companies for containing mine tailing reservoirs. Both the hydro-electric and mining industries have large interests in newer, more efficient methods of dam safety monitoring. A number of potential seepage areas were identified in several of the investigated dams.

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1. Introduction

The term self-potential is used to describe the naturally occurring electric fields that are found everywhere in the earth. The self-potential method is an applied geophysical method that involves measuring the potential differences that are created by these electric fields. The method has been in use since the early 19th century for applications including mineral prospecting, geothermal exploration and the investigation of groundwater movements. In recent years the method has enjoyed a renewed interest due to improvements in both the instrumentation and field methods. Today even very small self-potentials can be reliably measured.

The streaming potential phenomenon is the mechanism responsible for the selfpotentials generated by groundwater flows, and it is the only geophysical phenomenon directly related to the transport of subsurface water. Due to this fact, study of the streaming potential phenomenon has also enjoyed a renewed interest with respect to near-surface groundwater flow investigations. Such investigations can include monitoring geothermal activity, tracing contaminant plumes and monitoring the seepage through earth embankment dams.

The self-potential method is particularly useful for monitoring seepage in earth dams, and it has been successfully used to identify leakage zones for a number of investigated dams. In fact, the work presented in this thesis was originally inspired to some degree by the success of self-potential measurement in identifying leakage zones in the Suorva embankment dam on the Luleå River in northern Sweden (Triumf et al, 1995).

In 1998, there were an estimated 45,000 dams in the world with a height greater than 15 metres, and 70 percent of them were earth embankment dams (ICOLD, 1998). At 335 metres, the Rogun embankment dam on the Vakhsh River in Tadjikistan was the highest dam in the world. Canada's Syncrude Tailings dam was the largest at 540 million cubic metres of material. China's Three Gorges dam on the Yangtze River, begun in 1993 and expected to be completed in 2009, will become the world's largest and highest dam.

Sweden has 190 dams that classify as large, and of these, 130 are embankment dams. The highest dam in Sweden is the 125 metre high Trängslet embankment dam on the Dala River. The dam construction shown in Figure 1.1 is typical of those built by the Swedish hydropower industry (Statensvattenfallsverk, 1988). The central sealing core material generally comprises moraine and is protected from erosion by a layer of filter material. The core material is supported to either side by wide slopes of earth or rock fill. The number and thickness of the layers will vary from dam to dam.

Besides the hydro power industry, mining companies also build earth embankment dams in order to contain reservoirs for storing mine tailings. The reservoirs are typically located near the mine for convenience and thus sometimes require very long dams if the local topography is relatively flat. A similar construction to that used for hydro-electric dams is sometimes used; other times the dam is built as a composite mass of moraine and mining waste.



Figure 1.1. Schematic representation of a typical section through an earth embankment dam containing a central sealing core and slopes of support material.

Despite safety factors and precautions, dams do occasionally fail. Studies of past earth dam failures show three major causes: seepage and internal erosion in the embankment, seepage and erosion of the foundation and overtopping (ICOLD, 1995). With adequate surveillance, the first two causes can be detected and remedied before a failure occurs. The self-potential method is an ideal tool for monitoring seepage in embankment dams; yet even with past successes it is still too infrequently employed. However, with the increasing dependence on dams, their potential threat to human lives, infrastructure and the environment, combined with climate shifts and more recent dam failures, interest is again growing in the technique.

The principle goal of this thesis was to create a computer program to numerically model the self-potentials generated by liquid flows through an inhomogeneous, anisotropic three-dimensional domain. This would provide a valuable tool for gaining insight into the streaming potential phenomenon and provide information to aid in the interpretation of self-potential measurements. In order to achieve this, a study of the streaming potential phenomenon was undertaken together with a study of numerical modelling techniques and field measurements performed on a number of embankment dams.

In addition to being able to model the streaming potentials that are generated by flow in an embankment dam, the developed program could provide the electric potential distribution generated by any two-phased flow in a three-dimensional region of any shape. The program could thus be applied to any type of streaming potential investigation, or it could produce the electric potentials generated by other types of flow, such as heat.

1.1 Layout of the thesis

This thesis begins with relatively basic concepts and progresses toward more complex topics in the latter chapters. An attempt was made to follow a logical progression. The next two chapters cover the two electric geophysical methods that are central to this thesis and form a necessary knowledge base for subsequent chapters. Chapter 4 takes a closer look at the electrokinetic phenomena, including streaming potentials, and introduces many important concepts such as the electric double layer and coupled flow theory. Chapter 5 delves into the numerical modelling of streaming potentials

and the finite element method. The resulting computer program is showcased in Chapter 6, and the results of the field investigations are presented and discussed in Chapter 7. The thesis is brought to a close with a summary and discussion of the main results in Chapter 8, which also includes some ideas for future research.

2. The Self-Potential Method

The self-potential (SP) method involves the measurement and interpretation of the naturally occurring electrical potential differences that exist between any two points in the earth. These potential differences actually comprise two portions, one constant and unidirectional and the other fluctuating. The relatively steady part of the potential difference is the result of various electrochemical processes occurring in the ground, including streaming potentials; the time-varying potentials are due to magnetotellurics; i.e. fluctuations in the earth's magnetic field.

SP values generally range from a few tenths of a millivolt to several tens of millivolts, although values of several hundred millivolts can be observed. Such large values are often obtained over electrically conductive mineral deposits, over coal and manganese deposits, in areas of considerable topographic variation, in geothermal areas and in areas with high groundwater flow rates.

The SP method was first documented by Fox in 1830 for the investigation of sulphide veins in a Cornish mine. Systematic use of the method however didn't occur until after 1922 when Schlumberger introduced the use of non-polarizable electrodes. Since then the major environmental and engineering application of the SP method has been the investigation of subsurface water movements. Specific uses include the mapping of seepage flow through containment structures such as dams, dikes and reservoir floors; and the mapping of flow patterns in the vicinity of landslides, sinkholes, wells, shafts, tunnels and faults. The method has also been used to a lesser extent in mining exploration, geothermal investigations and for mapping chemical concentration gradients.

2.1 Origins of self-potentials

There are several different electro-chemical mechanisms that can create potential differences in the ground. Generally SP investigations are performed to help locate and delineate the potential sources associated with one or more of these mechanisms. This can however be difficult when several mechanisms are contributing anomalies as these will be superimposed and there is no certain way to distinguish by electro-chemical origin. The main mechanisms are briefly described below, for a more detailed description readers are referred to Parasnis (1986) and Friborg (1996). Despite wide speculation, these mechanisms are not fully understood.

2.1.1 Streaming potentials

Streaming potentials are a result of electric currents that are generated whenever an electrolyte moves with respect to a stationary solid that it is in contact with, as is the case when water flows through earth and rock. This was first observed in capillary tubes by Quincke in 1859, and a theoretical model for the effect in capillaries was later developed by Helmholtz in 1879. That model is still in use today and can be shown to be equally valid for flow through a porous medium.

Streaming potentials are caused by a mechanism known as electro-filtration, one of several electrokinetic phenomena discussed further in chapter 4. The basic theory of electro-filtration is that when a liquid moves with respect to a solid it is in contact with, it carries with it charged particles that are attracted to the charged surface of the solid, creating an electric convection current. This convection current will cause the mobile charges to deplete upstream and accumulate downstream, creating an electric potential difference. This potential difference is the streaming potential, which will in turn drive a conduction current back through the body of the fluid. In steady state these two currents will balance each other. The magnitude of the streaming potential depends on the resistance of the return current path, thus if the solid is not insulating, part of the conduction current will pass through it, reducing the streaming potential. The subsurface resistivity distribution will therefore play a large role in the shape and magnitude of streaming potential anomalies. This is discussed further in chapter 3.

Wherever groundwater is in motion, which is practically everywhere, there will exist SP anomalies due to streaming potentials. For the case of running water in contact with earth and rock the developed surface charge is typically negative, resulting in negative potential values upstream and positive values downstream. Naturally there is a correlation between topography and SP, with high points generally having negative SP anomalies.

Streaming potentials are very interesting in that they provide information directly related to subsurface flows. Other geophysical methods, such as the earth resistivity method, only provide secondary information about the effects of subsurface flows; however this secondary information can be of great assistance when interpreting SP measurements.

Of the various electro-chemical mechanisms that produce self-potentials, the streaming potential phenomenon is obviously the most important for groundwater investigations and it is discussed in greater detail in chapter 4. Streaming potential investigations have been performed to detect seepage in several dam studies, for example Butler et al (1990), Al-Saigh et al (1994), and Panthulu et al (2001). The first example details a comprehensive dam integrity investigation using a combination of several geophysical methods.

In order to be able to distinguish true streaming potential anomalies when interpreting SP measurements, an awareness of all of the possible SP-generating electrochemical mechanisms is required.

2.1.2 Sedimentation potentials

Sedimentation potentials are the result of the exact same mechanism that creates streaming potentials; however in this case solid particles move with respect to a liquid that is stationary as a whole. In principle, sedimentation potentials could occur where there are standing water bodies with high concentrations of suspended sediments; however the physical conditions required for generating significant SP anomalies would very rarely exist in nature. Sedimentation potentials are further discussed along with the other electrokinetic phenomena in chapter 4.

2.1.3 Mineral potentials

Mineral potentials occur above all kinds of electrically conducting mineral bodies and are probably the most common cause of strong SP anomalies. SP values caused by mineral bodies are typically greater than those associated with streaming potentials; although some comparatively high streaming potential anomalies have been observed, usually on hilltops.

Many theories have been developed in an attempt to explain the phenomenon. The earliest theories attributed oxidation of the ore body as the key mechanism. Later, Sato and Mooney (1960) pointed to flaws in these theories and developed a new model where electrons are lost in the lower portion of the ore body and gained in the upper part by a number of possible chemical reaction pairs and the ore body acts only as an electron conductor. The different electrochemical reactions at the upper and lower parts of the ore body create potential drops across the mineral-electrolyte interface that can be solved for by assuming chemical equilibrium.

A problem with this approach is that no current flow can exist under chemical equilibrium, thus no SP anomaly would be registered. The voltages at the interface are not only dependent on the chemical reactions, but also on the current flow, just as the voltage of a battery drops when current is drawn from it. The current flow is determined by the subsurface resistivity distribution, which therefore plays a large role in the magnitude of mineral potentials.

Kilty (1984) used non-equilibrium thermodynamic equations to expand the Sato and Mooney model. According to his model there are four separate voltages to consider: the potential drop in the ore body (V_o) , the potential drop in the ground and the interface voltages at the upper (V_u) and lower (V_l) parts of the ore body. The voltages can be related as $IR = V_u - V_l - V_o$ where I is the current flow and R is the resistance of the current path outside of the ore body. The mineral potential value is a part of the potential drop IR.

Because of the complex nature of the interactions between the electric current, the electro-chemical interface reactions and the subsurface resistivity distribution, it can be very difficult to predict the magnitude of SP anomalies caused by mineral potentials. The SP method has been used in the field of mining exploration for mineral prospecting investigations, for example Malmqvist and Parasnis (1972) and Logn and Bølviken (1974).

2.1.4 Diffusion potentials

Theoretically, if an excess of a certain type of ion were to exist at a point in the ground then diffusion forces would act to restore a homogenous distribution. The migration of the ions in the direction of the concentration gradient would constitute an electric convection current, which would in turn drive an electric conduction current in the reverse direction. This conduction current creates an electric potential drop that is the measured diffusion potential anomaly. The convection current can be calculated for a known concentration gradient; however things become much more complicated in nature where several different types of ion typically contribute to creating the diffusion current.

It is believed that concentration differences in the groundwater may contribute to background potentials encountered in most SP investigations; however their influence can be very difficult to determine. In addition, no suitable explanation exists for why diffusion potentials persist over time. By diffusion, the concentration differences should disappear over time, unless a continuous source of the excess ions existed. No continuous source of ions has been observed, although a redox reaction at the groundwater surface has been suggested.

2.1.5 Adsorption potentials

SP anomalies due to ion adsorption are known to occur above quartz and pegmatite granite bodies and are generally in the order of +20 to +40mV. This has been attributed to the adsorption of positive ions on the surface of these rocks (Semenov, 1974), but the electrochemical mechanism is not clear. The measured SP anomaly is the potential drop due to a current flow, thus the adsorption of a layer of static positive ions would not sustain the anomaly. Similar SP anomalies observed over clay deposits probably also belong in this category.

2.1.6 Thermoelectric potentials

Temperature gradients are also known to generate SP anomalies. Relatively large anomalies can be observed in geothermal areas, assumed to be caused by a combination of both electrokinetic and thermoelectric coupling (Corwin and Hoover, 1979). On the electrokinetic side, streaming potentials are created when thermal sources induce convection of the groundwater. The thermoelectric effect is not fully understood, but is believed to involve the differential diffusion rates of both ions in the groundwater and electrons and ions in the soil and rock. The thermoelectric coupling effect is usually expressed as a ratio between the temperature gradient and the resulting electric potential gradient. This ratio, called the thermoelectric coupling coefficient, has been shown to lie between -0.1 and 1.5 mV/°C for a variety of rock types (Nourbehecht, 1963). Interesting examples of the use of SP measurements for investigating geothermal areas are provided by Di Maio et al (1998) and Finizola et al (2004).

2.1.7 Vegetation

The occurrence of ground vegetation can lead to spurious potential anomalies, most likely due to its effect on soil moisture content and contributing streaming potentials. It is even possible that diffusion potentials occur around the roots of plants. The effects of vegetation can been seen as short wavelength SP anomalies with amplitudes of up to 150mV (Erntson and Sherer, 1986). It has also been observed that areas of dense vegetation tend to give positive SP values compared to areas of bare soil.

2.1.8 Precipitation

Precipitation will typically give rise to spurious SP anomalies due to streaming potentials. These potentials will misrepresent normal ground conditions and consideration should be made when taking measurements at times of heavy rainfall or high rate of snowmelt.

2.2 Field procedure

The magnitudes of SP anomalies associated with subsurface water movement are generally smaller than those associated with mineral and geothermal exploration, and the presence of man-made structures in the study area can create significant noise anomalies. It is therefore imperative that great care be taken in acquiring and interpreting SP data, and that the characteristic fields associated with artificial noise sources are recognised.

SP measurements are rather simple to perform, requiring only two electrodes, a voltmeter and the cables to connect them. There are two different field procedures for SP investigations: gradient and absolute potential. For the gradient method a dipole with a constant electrode separation (l) is moved along the survey area. If l is not too great then the ratio of the potential difference to length, $\Delta V/l$ measures the potential gradient. The absolute potential can be obtained by summing the potential differences along the profile; however the value obtained would contain the accumulated noise from each individual measurement. This can be reduced somewhat by 'leapfrogging', where the forward electrode becomes the rear electrode for the next measurement and only the rear electrode ever moves forward. Care must be taken in recording the polarity of each measurement when using this technique.

The absolute measurement method involves a stationary electrode and a roving electrode, connected by long cable reels. The stationary, or reference, electrode is usually placed outside of the study area in a spot where the SP values are expected to remain steady. The stationary electrode is connected to the negative terminal of the voltmeter and the mobile electrode is connected to the positive terminal. This method provides the absolute potential difference between the measurement points in the survey area and the stationary reference electrode.

Any voltmeter used in SP investigation should have a relatively high input impedance, at least 10^8 ohms, in order to prevent drawing appreciable current from the ground, which would disturb the potential distribution and cause polarisation of the electrodes. Most modern voltmeters have a suitably high input impedance.

Special non-polarizable electrodes should be used for SP measurements, although there are cases of simple metal stakes performing adequately (eg Butler et al., 1990). Usually the electrochemical reactions that occur where the metal meets ground moisture create potentials, called redox potentials, which may overshadow the self-potentials. This is discussed further below.

2.2.1 Electrode redox potentials

When a metal electrode is in contact with moisture in the soil, an electric potential difference will result due to electrochemical reactions at the electrode-electrolyte interface. The magnitude of the potential is difficult to determine as several different reactions are involved, depending on properties of both the electrode and the electrolyte. A study of the relation between self- and redox potentials has been performed by Timm and Möller (2001).

If an SP measurement were made between two identical electrodes placed at points with identical soil moisture conditions, the measurement would be unaffected. As this is rarely the case, however, it is strongly recommended that non-polarising electrodes be used for SP measurements. The metal of these electrodes is contained within a saturated salt solution of the electrode metal, which makes contact with the soil through a porous plug of wood or unglazed porcelain. A measurement between two such electrodes should represent the potential difference in the ground, as the metal of each electrode is in contact with the exact same type of saturated solution.

It has been shown, however, that the potential of a non-polarising electrode increases with increasing soil moisture content as well as with increasing temperature. Corwin (1989) reports an effect of 0.3 to 1 mV per percentage change in moisture content, and Kassel et al. (1989) describe an effect of 0.5 to 1 mV per degree of temperature change in the electrode's metal solution. These values were obtained for the most commonly used non-polarising electrodes, which are copper in a copper-sulphate solution (Cu-CuSO₄) and silver in a silver-chloride solution (Ag-AgCl).

The only way to compensate for the effects of soil moisture variations is to carefully observe the conditions during the investigation. The effects of temperature differences between electrodes can be compensated by performing regular drift calibrations. This involves recording the potential difference between electrodes when they are in a common electrolyte bath and applying the value to the measurements as a correction.

The fluctuating portion of SP measurements due to magnetotelluric effects can be accounted for by making regular measurements of the SP difference between two common reference points within the survey area. When using the absolute potential method, the stationary reference electrode is used as one of the reference points. The variations in this reference value over time are recorded and the remaining measurements are adjusted accordingly.

2.3 Interpreting SP data

There have been many attempts at quantitative interpretation based on theoretical anomalies calculated for simple geometric bodies located in a homogeneous half-space. Despite providing valuable insight into the streaming potential phenomenon, by disregarding the electric resistivity distribution this approach really only provides a qualitative understanding. Quantitative interpretation of SP anomalies can be achieved through numerical modelling techniques. The strength and geometry of the current sources that cause the anomalies are derived from knowledge of material properties and the driving forces, and the electric potential distribution is then calculated from this data.

For the case of streaming potentials the driving force is the flow of liquid caused by, for example, a hydraulic potential difference and the hydraulic and electric conductivities are two important material properties. The example of a flow through a porous earth dam is well approximated by the electric potential distribution caused by a positive and a negative current source at the outflow and inflow areas, respectively.

A major part of this thesis is devoted to the development of a numerical method for the quantitative interpretation of streaming potentials. The resulting finite element modelling program is presented in chapter 6 and the numerical formulations used in its development are discussed in chapter 5.

3. The Earth Resistivity Method

It has been established that the subsurface resistivity distribution will have a large influence on the form and magnitude of SP anomalies caused by most of the electrochemical mechanisms mentioned in the previous chapter. A fundamental knowledge of electric subsurface conditions should therefore be greatly beneficial for interpreting SP measurements. A common means of obtaining subsurface resistivity information is to use the earth resistivity method.

The earth resistivity method involves introducing a DC or low-frequency alternating current into the ground by means of two electrodes connected to a portable power source and measuring the resulting potential difference between a separate electrode pair. Simple metal stakes usually suffice for both current and potential electrodes, although non-polarizable electrodes are preferable for the potential electrode pair. The measurements provide information about the distribution of electric conductivity (σ) below the surface. Resistivity (ρ) is simply the reciprocal of electric conductivity ($\rho = 1/\sigma$) and represents a material's inability to conduct electric current. Obviously such information is of great value for geological investigations such as prospecting for oil, minerals or water.

3.1 Resistivity of rocks and minerals

The electric resistivity of natural rocks and sediments can vary greatly and depends on a number of factors. The amount and interconnectivity of various minerals will play a role. The resistivity of silicate minerals is typically very high ($10^6 \Omega m$ and up) whereas sulphides and most oxide minerals can be considered semiconductors with resistivities in the range of 10^{-6} to $10^{-2} \Omega m$. Graphite minerals also exhibit semiconductor properties with resistivities of the same order as sulphides.

The resistivity of water depends strongly on the concentration of salts, which provide dissolved ions that act as charge carriers. Fresh groundwater will usually have a resistivity in the range of 10 to 100 Ω m while saltwater will range from 100 to 1000 times more conductive (0.1 Ω m) (Thunehed, 2000). Most rocks act as insulators in a dry state; however, in nature they almost always contain some porewater, which will affect their bulk resistivity.

The degree of pore interconnectivity will greatly influence a rock's bulk resistivity, and the shape of the pores will also have a lesser effect. The bulk conductivity of sedimentary rocks can be estimated using Archie's law:

$$\sigma = \sigma_f f^m s^n, \qquad (3.1)$$

where σ_f is the electric conductivity of the fluid filling the pores, *f* is the porosity (volume fraction of porespace), *s* is the fraction of the porespace that is saturated, and *m* and *n* are dimensionless factors.

The value of m is dictated by the degree of cementation, roughly ranging from 1.3 for loose sediments to 2.2 for well-cemented ones. The value of n is usually around 2.0 but can be much greater when less than 30% of the porespace is saturated. Equation 3.1 works best for sedimentary rocks with a porosity of a few percent or more and lacking any clay minerals.

The influence of water-filled pores and micro-fissures is usually less than that of larger water-filled fractures and fissures in the rock. The frequency and size of the fissures combined with the porewater conductivity will greatly affect a rock's bulk resistivity. Additionally, the direction of the fissures typically contributes to the anisotropy of the resistivity, i.e. the resistivity is greater perpendicular to the direction of the fissures. A simple model to estimate the resistivity of fissured rock, where the fissures all run parallel, has been developed by Stesky (1986).

The resistivity of rock is also known to vary with pressure and temperature. Studies have shown that resistivity increases with increased pressure; the reason is assumed to be the closure of fissures and fractures at higher pressures. This should be kept in mind when measuring resistivities at considerable depth and when comparing in-situ measurements with laboratory data.

The conductivity of practically all rocks and minerals will increase with increasing temperature according to $\sigma = \sigma_0 e^{-E/kT}$, where *E* is the mineral's activation energy, *k* is Boltzmann's constant and *T* is the absolute temperature. Additionally the conductivity of electrolytes such as water increases with increasing temperature due to lower viscosity providing greater ion mobility. This effect should not be neglected when investigating areas with geothermal activity.

3.2 Apparent resistivity

Ohm's law defines the behaviour of an electric current (I) in a linear conductor of uniform cross-section as

$$I = -\frac{dV}{R}, \qquad (3.2)$$

where dV is the potential difference between the conductor ends and R is the resistance of the conductor. The resistance is directly proportional to the conductor's length (dl) and inversely proportional to its cross-sectional area (a) such that

$$R = \rho \frac{dl}{a}.$$
 (3.3)

Note that the resistivity is a proportional constant that depends on the conductor material, whereas the resistance is a property of the path that the current takes through the conductor. Equations 3.2 and 3.3 can be combined to give

$$\frac{I}{a} = -\frac{1}{\rho} \frac{dV}{dl} \text{ or } j = \frac{1}{\rho} E, \qquad (3.4)$$

where j = I/a is known as the current density (A/m²) and E = -dV/dl is called the electric field (V/m).

When a point current electrode is placed on the surface of a homogeneous isotropic half-space then the current spreads through the half-space symmetrically and is equal at all points that are the same radial distance (r) from the electrode. This allows equation 3.4 to be rewritten replacing dl with dr and a with the surface area of a hemisphere $(2\pi r^2)$. Integrating the equation would then provide the potential (V) at a distance (r) from the point current electrode:

$$V(r) = \frac{I\rho}{2\pi r} \frac{1}{r} + C, \qquad (3.5)$$

where *C* is an arbitrary constant that would become zero if *V* is assumed to be zero at an infinite distance from the electrode. There are of course two current electrodes in practice, a positive sending electrode (*A*) and a negative receiving electrode (*B*). The electric potential at any point in the half-space would then be

$$V = \frac{I\rho}{2\pi} \left(\frac{1}{r_A} - \frac{1}{r_B} \right), \tag{3.6}$$

where r_A and r_B are the distances from the point to the positive and negative electrodes respectively.

If *M* and *N* are the positive and negative potential electrodes, respectively, then the potential difference between *M* and $N(\Delta V)$ can be calculated from equation 3.6 as

$$\Delta V = \frac{I\rho}{2\pi} \left(\frac{1}{AM} - \frac{1}{BM} - \frac{1}{AN} + \frac{1}{BN} \right), \tag{3.7}$$

where AM, BM, AN and BN represent the distances between the respective electrodes.

In the ER method the potential difference between M and N is measured and equation 3.7 is used to calculate the resistivity according to the electrode configuration used. The resistivity calculated is referred to as the apparent resistivity (ρ_a) because equation 3.7 assumes a homogeneous isotropic half-space, which is most often a crude approximation of true subsurface conditions. The apparent resistivity can be considered a weighted average of all the resistivities encountered in the measurement.

3.3 Field procedures

A number of different methods exist for measuring electric resistivities in the earth. The most common methods involve placing two metal stakes in the ground and connecting them to a portable current source. The current is briefly switched on and the potential difference is measured across two potential probes also placed in the ground. Exactly how the current and potential electrodes are placed on the ground determines which areas of the subsurface most influence the measurement.

3.3.1 Electrode configurations

A variety of different electrode configurations exist for performing resistivity measurements. The type of information required and prevailing geological conditions will influence the selection. Following is a brief description of the most common configurations in use.

A Wenner configuration is achieved by placing the four electrodes inline with the potential probes between the current electrodes (in the order *AMNB*) and with a common spacing between them (AM = MN = NB = s). For this configuration, equation 3.7 reduces to

$$\rho_a = 2\pi \cdot s \frac{\Delta V}{I} \,. \tag{3.8}$$

The Schlumberger electrode configuration differs from the Wenner configuration only in that the distance between the two potential electrodes (MN = 2l) is kept much smaller than the distance between the two current electrodes (AB = 2L). For the Schlumberger array, equation 3.7 can be reduced to

$$\rho_a = \frac{\pi L^2}{2l} \frac{\Delta V}{I}.$$
(3.9)

A bipole-bipole or dipole-dipole array is created by separating the current electrode pair from the potential electrode pair. Typically both pairs will have a common spacing but they can be placed anywhere with respect to each other. For example the pairs could be placed inline, parallel, perpendicular or at any angle to each other.

A pole-dipole array is produced by connecting one of the current electrodes to a long cable reel and placing it far from the array, i.e. at infinity. If s is the potential electrode spacing MN and the current pole is placed inline with and at n spacings from the potential electrode pair, then equation 3.7 would reduce to

$$\rho_a = 2\pi \cdot n(n+1) \cdot s \frac{\Delta V}{I}.$$
(3.10)

Of course a pole-pole configuration is also possible where one of the potential electrodes is also moved to infinity, leaving only one current pole and one potential pole.

3.3.2 Apparent resistivity mapping

Resistivity mapping or profiling involves moving an electrode array along profile lines crossing the study area to investigate two- or three-dimensional structures in the ground. A Wenner or dipole-dipole configuration is typically used and the electrode spacing is selected based on the desired depth of investigation. The spacing is typically kept constant, providing a lateral resistivity distribution at a constant depth along the profiles. This method can be labour intensive when a large spacing is required as each measurement requires the displacement of all four electrodes.

3.3.3 Vertical sounding

A vertical electric sounding provides the apparent resistivity variation with depth for a horizontally layered earth. This is achieved by taking a number of measurements at a common point with successively larger electrode separations. Dipole-dipole and Schlumberger arrays are typical for electric soundings because they simplify the field procedure by requiring only two of the four electrodes to be moved as the array is expanded. For the dipole-dipole array the current electrode pair is moved successively further from the potential pair, or vice-versa, and for the Schlumberger array the current electrodes. Measurements can be further simplified by using a pole-dipole or polepole configuration, thus requiring only one electrode to be moved. It is advisable to perform a sounding at a point in two orthogonal directions to check for the influence of two- and three-dimensional structures.

The calculated resistivity values can be plotted as a function of the electrode separation to produce a sounding curve. If the earth is assumed to consist of plane homogenous layers, it is fairly simple to calculate a theoretical sounding curve. A theoretical model that approximates the subsurface conditions can be obtained by assuming the number of layers and adjusting their thickness and resistivity values so that the model's sounding curve approaches the measured curve. Computer software could be used to quickly optimise the selected model.

A special condition occurs when a highly conductive layer lies between two relatively resistive layers. This condition can lead to ambiguities in the interpretations. The conductive layer will attract most of the current flow and, with a thickness of *h* and resistivity of ρ , it's resistance to this flow would be $R = \rho \Delta l/(h\Delta w)$ for an arbitrary block of length Δl and width Δw . Thus all layers with the same ρ/h ratio would be electrically equivalent.

3.3.4 Apparent resistivity pseudosections

Electric pseudosections are the result of combining the two previous methods, providing both lateral and vertical resistivity variations. The electrode array is both expanded, to obtain information from greater depth, and moved along a profile line, to provide lateral information. As a result, the method is relatively slow, even when specialised equipment is used to simplify the procedure.

The Wenner and dipole-dipole arrays are well suited to pseudosections due to their use of common electrode spacing. Typically a number of electrodes are placed along the survey line at multiples of a common spacing and are all connected to a switchboard that is in turn connected to a portable computerised voltmeter/power source. A number of measurements corresponding to different depths are then made via the switchboard and software installed on the computerized unit. The software determines which electrodes act as A, B, M and N respectively for each measurement. In this way several sounding and mapping points can be taken with multiple

connected electrodes rather than moving single electrodes repeatedly. When the measurements are complete the entire array is moved a distance along the survey line and the measurements are repeated, as in the mapping method.

The number of electrodes that are connected will affect the number of measurements that are possible with each setup as well as the time required to move between each set of measurements. The spacing used between the electrodes determines the relative depth of the investigation and will also affect the time required to move between measurements. Pole-dipole and pole-pole arrays are also possible; however the cable reels connecting the removed pole(s) must be carried along the survey line with the rest of the array. It should also be kept in mind when using either of these arrays for pseudosections that they will produce asymmetric resistivity anomalies over symmetrical structures due to their asymmetrical nature.

The acquired measurement data is typically converted to apparent resistivity values according to equation 3.7 and presented as a vertical section with apparent resistivity contours. Each measurement point is assigned a pseudo position in the section based on its electrode position (x) and spacing (z). A number of computer modelling programs exist that can provide subsurface resistivity models for the obtained measurement pseudosections.

The forward modelling technique used to interpret vertical soundings can also be used to interpret pseudosections. Another method called inversion modelling, however, requires no assumptions of the subsurface geometry. This method involves dividing the section into small elements and calculating the resistivity each element would require to produce the measured section. The elements can be either two or three dimensional, where the 2D elements are assumed to extend infinitely in the plane perpendicular to the section.

3.4 Influence of the resistivity distribution on SP

The resistivity distribution in the ground will play an important role in the shape of any measured SP anomalies. Such anomalies are caused by one or more of the electro-chemical mechanisms described in chapter 2. Each of these mechanisms can be thought of as an electric current source with a set geometry and strength, emphasising the fact that a measured SP anomaly is a potential drop due to current flow in the ground. Seen in this light it is clear that the resistivity distribution will influence SP measurements.

For a homogeneous isotropic ground only the geometry of the current sources will determine the shape of the SP anomaly; however the existence of any resistivity variations will distort the shape of the anomaly. An example of this is presented in Figure 3.1, where SP anomalies are shown for a point current source in a homogeneous half-space on the left and for the same point source in the vicinity of a vertical contact on the right. The vertical contact separates two zones that differ in electric resistivity by a factor of ten. The resistivity difference clearly influences the shape of the SP anomaly; an effect that would be difficult to explain without resistivity data. From this, the importance of the resistivity distribution in the ground

becomes apparent and any such information can be of considerable assistance when attempting to interpret SP measurements.



Figure 3.1. The effect of resistivity variation on SP anomalies. An SP anomaly generated by a point current source in a homogeneous half-space (A), and the anomaly generated by the same source in the vicinity of a vertical contact (B). The source is located at x = -6 m, y = 0 m and the dashed line at x = 0 m indicates the vertical contact. The ratio of the resistivity values across the contact is 10.

4. Streaming potentials and other electrokinetic phenomena

As mentioned in chapter 2, streaming potentials are the most interesting electrochemical source of SP anomalies for ground flow investigations. In this chapter the electrokinetic phenomena that is the cause of streaming potentials is discussed in greater detail.

Streaming potentials are caused by electro-filtration; one of four known electrokinetic phenomena discovered in the early 19th century. The first two phenomena were discovered by Reuss in 1809 when he discovered that applying an electric field could make both water flow through a sand-filled tube and clay particles move in a water-filled container (Dukhin and Derjaguin, 1974). The first event, the transport of a liquid relative to a stationary solid phase as a response to an applied electric field, is called electro-osmosis. The second event, the transport of solid particles in a liquid that is stationary as a whole as a response to an applied electric field, is known as electrophoresis.

The two remaining electrokinetic phenomena are the converse effects of electroosmosis and electrophoresis. The opposite of electro-osmosis is electro-filtration, or the streaming potential, which is the occurrence of an electric field in response to the transport of liquid relative to a stationary solid phase. This was first recorded by Quincke in 1859. The opposite of electrophoresis is called a sedimentation potential, which is the occurrence of an electric field in response to the movement of solid particles in a stationary liquid, discovered by Dorn in 1880.

4.1 The electric double layer

The driving mechanism behind all four electrokinetic phenomena is the development of a surface charge on a solid phase in contact with a liquid. The surface charge is usually caused by chemical interactions between the solid and liquid phases. Most minerals only develop a surface charge when in contact with an electrolyte; however some, such as clay particles, have a permanent unbalanced surface charge due to imbalanced crystal structures. The surface charge of the solid phase will attract charges of the opposite sign in the liquid to the interface, creating a diffuse layer of counter-ions next to the surface of the solid. This redistribution of charge along a solid-liquid interface is known as the electric double layer and is the key to understanding the electrokinetic phenomena.

When the liquid and solid phases are moved in relation to each other, a small layer of the liquid will remain attached to the solid. A shearing plane is therefore created within the liquid, located a small distance from the surface of the solid. The shear plane will lie somewhere within the diffuse layer of counter-ions. Because of this, the relative movement between solid and liquid will cause the transport of some of the charges, creating an electric current. The opposite is also true; that the transport of some of the charges by an applied electric current will cause relative movement between the solid and liquid phases. This is the underlying principle for all four of the electrokinetic phenomena. Applying an electric field to the solid-liquid interface will cause electrostatic forces to act on the charges in the electric double layer. The electrostatic forces will act on the charges so that they move in the direction of the applied field. If the solid phase is stationary, then the counter-ions in the liquid will move with respect to the solid. Through internal friction, the liquid will follow the motion of the charged particles and move with respect to the solid, creating electro-osmosis. Conversely, should the liquid phase be stationary then the electrostatic forces would cause solid particles to move with respect to the liquid, causing electrophoresis.

For the cases of streaming potentials and sedimentation potentials it is mechanical forces such as pressure gradients that cause the relative movement between the solid and liquid phases. Consider an electrolyte flowing parallel to the solid-liquid interface, as shown in Figure 4.1. Some of the counter-ions in the diffuse part of the double layer would be sheared off and transported with the flow. This will result in a surplus of the counter ions at the downstream end of the flow and a lack of them at the upstream end. This charge separation creates an electric potential difference that in turn drives a return current in the opposite direction through the main body of the electrolyte. The streaming potential is the electric potential difference at equilibrium. Conversely, when solid particles are moved by mechanical forces in a stationary liquid, the same mechanism operates to produce sedimentation potentials.



Figure 4.1. Schematic representation of an electrolyte flowing through a stationary porous medium with a close-up of the electric double layer. As the liquid flows some of the counter ions in the diffuse portion of the double layer will be sheared off and carried along with the fluid, creating an electric current.

4.2 The zeta-potential

Obviously the electric double layer is a very important factor for electrokinetic phenomena and therefore any information that can aid in quantifying its properties is useful. A fundamental physical property of the double layer is the zeta-potential (ζ), which is defined as the electric potential at the shear plane in the electrolyte. Although not directly measurable, the zeta-potential can be calculated from surface charge measurements or estimated from electrokinetic measurements. If the former method is used, the results are dependent on the assumption used for the potential distribution in the electrolyte.



Figure 4.2. A simplified model representing the charge distribution at the solid-liquid interface, corresponding to the Gouy-Chapman model. Only the excess charges are shown, otherwise the liquid can be assumed to contain equal amounts of positive and charges. negative In this example the surface charge is assumed to be negative. Below is a gualitative desthe potential cription of distribution in the diffuse part of the double layer. The vertical dashed line indicates the plane of shear. The zetapotential is the electric potential value at the shear plane.

A simple model of the potential distribution in the electric double layer is illustrated in Figure 4.2. The shear plane occurs at the point where the liquid begins to flow, a small distance from the solid surface. By definition, the zeta-potential is the electric potential at this point. This model was developed independently by Gouy in 1910 and Chapman in 1913 (Dukhin and Derjaguin, 1974). More complex models exist for the potential distribution in the double layer (eg Stern, 1924); however it will be shown below how the actual form of the distribution does not affect the streaming potential.

The zeta-potential is directly proportional to the surface charge that forms on the solid when it is in contact with an electrolyte. Unless the solid material already possesses a surface charge due to an imbalanced crystal structure, the two main chemical processes that act to develop a surface charge are the adsorption of ions and the hydrolysis of surface hydroxyl groups. Both of these processes typically occur simultaneously and are dependant on the chemical compositions of both the electrolyte and the solid. The chemistry at the interface can be treated by general chemical equilibrium equations and the equilibrium constants for many materials can be found in chemistry texts.

Of the solid and liquid chemical compositions, the pH value of the electrolyte is the property that has the most effect on the surface charge and hence the zeta-potential. The pH value has a large effect on the hydrolysis of surface hydroxyl groups and a lesser one on the adsorption of ions. Using chemical equilibrium equations, the surface charge density can be shown to decrease with increasing pH (Stumm, 1992). The corresponding influence on the zeta-potential is that it will also decrease with increasing pH.

The ionic strength of the electrolyte is another property of its chemical composition that affects the zeta-potential by its influence on the thickness of the double layer. Ionic strength is a measure of the amount of charged particles in the electrolyte and is defined as

$$I = \frac{1}{2} \sum c_i z_i^2,$$
 (4.1)

where c_i is the number of charged particles of type *i* and charge z_i . A higher ionic strength will act to compress the diffuse part of the double layer against the solid, reducing its thickness and consequently the zeta-potential.

The pH value of the electrolyte can also have an additional influence on the zetapotential by affecting the ionic strength. At very high and very low pH values, there will be high concentrations of negative and positive charges, respectively. These high concentrations will significantly contribute to increasing the ionic strength of the electrolyte, thereby reducing the zeta-potential.

Temperature is another factor that can influence the zeta-potential. Its effect can be quite complex as it is known to influence all of the chemical processes that contribute to the formation of the surface charge. The equilibrium constants for various materials and electrolytes are temperature-dependant. Temperature will also affect the thickness of the double layer. At higher temperatures, the charged particles will have increased mobility, expanding the double layer and increasing the zeta-potential. The raised temperature can however simultaneously act to reduce the zeta potential by reducing the adsorption of ions, for example. The simultaneous contribution of several temperature-dependant mechanisms makes the overall temperature effect very difficult to predict.

4.3 Streaming Potentials

As previously mentioned, streaming potentials occur when a liquid that is in contact with a solid is put in motion by mechanical forces. The electric double layer that forms at the solid-liquid interface causes the transport of excess charges along with the fluid as it flows along the solid surface. This transport of charges constitutes an electric convection current that results in a surplus of the charges downstream and a deficit upstream. This potential difference is the streaming potential.

The streaming potential will cause an electric conduction current to flow in the reverse direction through both the liquid body and the solid matrix, should it be conductive. The conduction current acts to cancel the convection current in steady state. If the solid material is electrically conductive to any degree, the magnitude of the streaming potential be will decreased. As discussed in chapter 3, this is the case for most rocks and minerals found in nature.

4.3.1 Flow through a capillary tube

The relationship between the electric potential difference and a fluid pressure difference can be demonstrated with the simple example of flow through a capillary tube. This example can then be broadened to encompass flow through porous media.

For the case of a capillary tube the surface charge develops on the inside of the tube wall and the convection current is carried with the fluid through the tube. Pouiseille's equation (Lamb, 1945) defines the axial velocity of a liquid driven by a pressure difference (ΔP) through a capillary tube of radius *r* and length *l* as

$$v(r_{c}) = \frac{\Delta P}{4\eta l} (r^{2} - r_{c}^{2}), \qquad (4.2)$$

where η is the dynamic viscosity of the liquid and r_c is the distance from the centre of the tube. In reality r is not the radius of the tube but the distance from its centre to the shear plane.

If $\rho(r_c)$ is the density of the excess charges that build the convection current, then the axial convection current density can be defined as

$$j_{conv} = v(r_c) \cdot \rho(r_c). \tag{4.3}$$

The excess charges that are attracted to the surface charge of the tube will decrease sharply with increasing distance from the tube wall; therefore only values of r_c that are close to r will significantly influence this equation. This permits some simplification of equation 4.2 by expanding the term $(r^2 - r_c^2) = (r + r_c)(r - r_c)$. Replacing r_c with r in the first parenthesis and setting $(r - r_c) = x$ results in an approximate axial velocity of

$$v(x) \approx \frac{r x \Delta P}{2\eta l}, \qquad (4.4)$$

where x is the distance from the capillary wall. The total convection current is obtained by integrating equation 4.3 over the cross-sectional area (A) of the capillary tube. Inserting equation 4.4 into the integral yields

$$I_{conv} = \int_{A} j_{conv} dA = \int_{0}^{r} 2\pi r_{c} v(r_{c}) \rho(r_{c}) dr_{c} = -\int_{r}^{0} \pi (r-x) \frac{r x \Delta P}{\eta l} \rho(x) dx .$$
(4.5)

Because the excess charges that form the current are located near the capillary wall, a further simplification of $r - x \approx r$ can be made, yielding a convection current of

$$I_{conv} = -\frac{\pi r^2 \Delta P}{\eta l} \int_{r}^{0} x \rho(x) dx. \qquad (4.6)$$

The relationship between electric charge density, permittivity (ε) and electric potential (ϕ) is defined by Poisson's equation:
$$\nabla^2 \phi = -\frac{\rho}{\varepsilon}, \quad \text{where} \quad \nabla^2 \phi = \frac{d^2 \phi}{dx^2} + \frac{d^2 \phi}{dy^2} + \frac{d^2 \phi}{dz^2} \tag{4.7}$$

for a three-dimensional case. For the one-dimensional case of a capillary tube the charge density becomes

$$\rho(x) = -\varepsilon \frac{d^2 \phi}{dx^2}.$$
(4.8)

Substituting this into equation 4.6 and integrating by parts results in a convection current of

$$I_{conv} = \frac{\pi r^2 \Delta P}{\eta l} \varepsilon \left[\left(x \frac{d\phi}{dx} \right)_{x=r}^{x=0} - \int_r^0 \frac{d\phi}{dx} dx \right] = -\frac{\pi r^2 \Delta P}{\eta l} \varepsilon \int_0^\zeta d\phi = -\frac{\pi r^2 \Delta P}{\eta l} \varepsilon \zeta .$$
(4.9)

The first term in the square brackets becomes zero because the potential gradient $(d\phi/dx)$ is zero at the centre of the tube (x = r). The second term in the square brackets becomes the potential difference between the centre where $\phi = 0$ and the shear plane where $\phi = \zeta$, which is by definition the zeta-potential. The final solution of equation 4.9 shows that the convection current is independent of the detailed electrical structure of the double layer.

The negative sign in equation 4.9 reflects the opposite polarity of the zeta potential value with respect to the excess charges in the diffuse part of the double layer. The simplifications used in deriving equation 4.9 do not strictly hold over the entire integration range; however the excess charge density ($\rho(x)$) reduces to zero at a small distance from the capillary wall, thus reducing the integral to zero for values of x where the simplifications don't hold.

The convection current creates the potential difference that is the streaming potential. This potential difference drives a conduction current in the reverse direction, which can be defined by Ohm's law as

$$I_{cond} = \frac{\Delta V}{R},\tag{4.10}$$

where ΔV is the streaming potential difference and *R* is the resistance of the current path. For the case of a capillary tube, the return current can flow through the liquid body and along the surface of the capillary walls, yielding a resistance of

$$\frac{1}{R} = \frac{\pi r^2}{l} \sigma_f + \frac{2\pi r}{l} \sigma_s, \qquad (4.11)$$

where σ_f and σ_s are the electric conductivities of the fluid and the capillary surface respectively. In steady state the conduction and convection currents will cancel each

other such that $I_{conv} + I_{cond} = 0$. From this, equations 4.9 to 4.11 can be combined to yield

$$\frac{\Delta V}{\Delta P} = \frac{\varepsilon \zeta}{\eta \left(\sigma_f + 2\frac{\sigma_s}{r}\right)} = S, \qquad (4.12)$$

where *S*, defined as the ratio of the electric potential difference to the pressure difference, is known as the streaming potential coefficient. When surface conduction is negligible, the second term in the parenthesis is omitted and the streaming potential coefficient becomes independent of the radius of the capillary. The contribution of surface conduction is only significant with capillaries of very small diameter or fluids of very low conductivity.

4.3.2 Flow through a porous medium

A porous medium can involve anything from a sediment-filled tube to a solid, porous rock mass. A liquid driven by a pressure difference to flow through a porous medium will produce a streaming potential in the same manner as in a capillary tube. The geometry of the flows becomes significantly more complex than for a capillary tube; however Overbeek (1952) has shown that equation 4.12 applies equally to the case of a porous medium provided surface conduction is negligible.

Another important assumption is that the convection and conduction currents share the same geometry; for when this is the case, then any geometry terms in their formulations will cancel each other when combined in the steady state equation. Assuming the solid matrix to be electrically insulating will therefore always result in equation 4.12 without the surface conduction (σ_s) term. This can be shown by a simple example.

If the pore space of a porous medium were approximated by N capillaries per unit of area, the convection and conduction currents would be the sum of the respective currents in each of the capillaries. From equations 4.9 and 4.10 the currents would become

$$I_{conv} = -\frac{\varepsilon \zeta}{\eta l} \Delta P \sum_{i=1}^{N} \pi r_i^2 \quad \text{and} \quad I_{cond} = \frac{\sigma_f}{l} \Delta V \sum_{i=1}^{N} \pi r_i^2 , \qquad (4.13)$$

where r_i is the radius of each capillary. In steady state $I_{conv} + I_{cond} = 0$, resulting in the geometry terms cancelling each other and a streaming potential coefficient of

$$S = \frac{\Delta V}{\Delta P} = \frac{\varepsilon \zeta}{\eta \sigma_f},\tag{4.14}$$

which is a simplified version of equation 4.12. If the conduction current wasn't solely limited to the fluid in the capillaries, then the bulk conductivity of the porous media (σ_b) could be used in place of the fluid conductivity and the entire area would replace

that of the capillaries. The conduction current for a unit area of the porous media would thus become

$$I_{cond} = \frac{\sigma_b}{l} \Delta V, \qquad (4.15)$$

and the steady state condition would result in a streaming potential coefficient of

$$S = \frac{\Delta V}{\Delta P} = \frac{\varepsilon \zeta}{\eta \sigma_b} \sum_{i=1}^N \pi r_i^2 .$$
(4.16)

The sum of the cross-sectional areas of the capillaries in this equation is in fact the porosity (f) of the porous medium. By rewriting equation 4.16 as

$$S = \frac{\Delta V}{\Delta P} = \frac{\varepsilon \zeta}{\eta \sigma_b} f , \qquad (4.17)$$

it can be seen that the bulk conductivity of the porous medium and the fluid conductivity are related as

$$\sigma_b = \sigma_f f , \qquad (4.18)$$

which is a simplified version of Archie's law that was discussed in chapter 3. This would indicate that should the bulk conductivity be used in equation 4.14 rather than the conductivity of the fluid, the streaming potential coefficient would be overestimated by a factor of approximately 1/f.

The ratio of the fluid conductivity to the bulk conductivity is commonly referred to as the formation factor (F). According to equation 3.1 the formation factor is related to the porosity as

$$F = \frac{\sigma_f}{\sigma_h} = f^{-m} s^{-n} \,. \tag{4.19}$$

When the solid matrix is non-conductive the formation factor is closely related to the permeability of the medium. A higher permeability results in a lower formation factor.

Although equation 4.14 is only strictly correct when surface conductivity and the conductivity of the solid matrix are negligible, this only becomes a problem when attempting to calculate the zeta-potential from a measured streaming potential coefficient. Should the solid matrix be conductive, part the conduction current will flow through it and the streaming potential will be reduced. This effect will be included in any measurements of the streaming potential coefficient, and thus the measured values can be directly used for modelling purposes.

4.3.3 The coupled flow formulation

The theory of coupled flows is a thermodynamic theory that can be used to relate different flow phenomena. The generalised form of the equation describing coupled flows is

$$J_i = \sum_{i=1}^n C_{ij} X_j , \qquad (4.20)$$

where J_i is the flow density of type *i*, X_j is a driving force of type *j* and C_{ij} is a coupling coefficient that relates flow of type *i* to a force of type *j*. Using this equation to account for all possible coupled flows would be very difficult; therefore the effects of some driving forces are assumed to be negligible. For the case of streaming potentials the hydraulic and electric gradients are of interest and chemical and thermal gradients are assumed to be zero. This results in the following two equations:

$$\mathbf{Q} = -C_{11}\nabla\xi - C_{12}\nabla\phi \quad \text{and} \tag{4.21}$$

$$\mathbf{J} = -C_{21}\nabla\xi - C_{22}\nabla\phi, \qquad (4.22)$$

where **Q** is the liquid flow density (volume flow per unit area), **J** is the electric current density, $\nabla \xi$ is the pressure potential gradient, $\nabla \phi$ is the electric potential gradient and C_{ij} are the coupling coefficients. C_{11} relates fluid flow to a pressure gradient and becomes the hydraulic conductivity (*K*); similarly C_{22} relates current flow to a potential gradient and becomes the bulk conductivity (σ_b). C_{12} and C_{21} are cross-coupling conductivities that relate fluid flow to a potential gradient (electro-osmosis) and current flow to a pressure gradient (streaming potential), respectively. These two conductivities are actually equal $C_{12} = C_{21} = C$ according to Onsager reciprocal relations. This can be shown by thermodynamic deliberation (Yeung and Mitchell, 1993).

If the cross-coupling terms in equations 4.21 and 4.22 were neglected then only the direct flow terms on the diagonal would remain and equation 4.21 would reduce to Darcy's law and equation 4.22 would take the more familiar form of Ohm's law.

A limitation of the coupled flow theory is that the flows are assumed to be linearly related to each driving force. This has been proven to be the case within wide limits by experiments; however this should be kept in mind when large flows or gradients are involved. Another assumption made for this method is that the medium is locally homogeneous, which is a simplification. This simply means that average values of the coupling coefficients are used rather than separate values for the pores and the matrix, resulting in average flow values.

Within a homogeneous area of a porous medium it is clear that the total current density must be zero at all points and not just when integrating over the whole medium. Setting J = 0 into equation 4.22 results in

$$\frac{\nabla\phi}{\nabla\xi} = -\frac{C}{\sigma_b}.$$
(4.23)

For a volume of one unit area by length *l*, the potential and pressure gradients can be approximated by $\nabla \phi = \Delta V/l$ and $\nabla \xi = \Delta P/l$ respectively. This allows a comparison with equation 4.14, yielding a streaming potential coefficient of

$$S = -\frac{C}{\sigma_b} = \frac{\varepsilon \zeta}{\eta \sigma_f} \,. \tag{4.24}$$

Recalling that σ_f / σ_b is the formation factor, the cross-coupling conductivity would thus have the form

$$C = -\frac{\varepsilon\zeta}{\eta F} = -\frac{\varepsilon\zeta}{\eta} f^m s^n.$$
(4.25)

Because the formation factor tends to decrease with increasing permeability, it becomes evident from equation 4.25 that the cross-coupling coefficient increases with increasing permeability. This makes sense as a higher flow rate should generate a larger convection current.

The cross-coupling conductivity plays a role equally as large as those of the hydraulic and electric conductivities when using coupled flow theory to solve streaming potential problems. Although several experimental studies have been performed to obtain streaming potential coefficient values, very few of them represent realistic geological conditions. Most of these studies were made using pure minerals in order to investigate the zeta-potential. An exception from this is Friborg (1996), who performed laboratory measurements on a number of samples chosen to represent real in-situ geological conditions. A summary of his results are presented in Table 4.1. Included in his work is a summary of previous experimental studies.

The theory of coupled flows is well-suited to numerical modelling and was used to develop the finite element program for modelling the streaming potential phenomenon. How the program was formulated is covered in the next chapter and various aspects of the program itself are presented in chapter 6.

Table 4.1. Summary of experimental values for a number of geological soil samples from northern Sweden. Pb-1,3 and Gv are well-sorted sandy tills, Pb-2 is a silty till, QS is a quartz sand used as a control material and Sd is a till sample taken from the core material of the Suorva earth dam. k_{10} represents the grain size that 10% of the sample is finer than. To obtain the cross-coupling coefficient (*C*), simply divide $-S/\rho_f$ by the formation factor (*F*). * represent unstable values. After Friborg, 1996.

Sample	<i>K</i> (m/s)	k ₁₀ (mm)	ρ _f (Ωm)	T (°C)	рН	F	S (mV/kPa)	S/ρ _f (mV/kPaΩm)
Pb-1a	2.1·10 ⁻⁴	0.27	124	7.9	7.6	3.5	-0.37	-3.0·10 ⁻³
Pb-1b		0.27	129	8.2		3.5	-0.40	-3.1·10 ⁻³
Pb-1c		0.27	124	8.4		3.5	-0.35	-2.8·10 ⁻³
Pb-2a1	5.4·10 ⁻⁶	0.06	118	18.6		5.4	-1.79	-1.5·10 ⁻²
Pb-2a2		0.06	120	17.6	6 [*]	5.4	-0.94	-7.8·10 ⁻³
Pb-2b1	5.1·10 ⁻⁶	0.06	95	18.9	5.6	5.4	-0.74	-7.8·10 ⁻³
Pb-2b2		0.06	98	17.8	6.0	5.4	-0.69	-7.0·10 ⁻³
Pb-2b3		0.06	100	17.3	6.0	5.4	-0.67	-6.7·10 ⁻³
Pb-2c1		0.06	101	18.8	6.6	5.4	-0.89	-8.8·10 ⁻³
Pb-2c2		0.06	100	18.5	6.6	5.4	-0.89	-8.9·10 ⁻³
Pb-2c3		0.06	102	18.6	6.3	5.4	-0.92	-9.0·10 ⁻³
Pb-2c4		0.06	102	17.4	6.4	5.4	-0.90	-8.8·10 ⁻³
Pb-2c5		0.06	108	16.7	6.6	5.4	-0.86	-8.0·10 ⁻³
Pb-3a1	9.6·10 ⁻⁴	0.35	89	24.5	7.2	8.4	-0.64	-7.2·10 ⁻³
Pb-3a2		0.35	71	29.5	7.4	8.4	-0.63	-8.9·10 ⁻³
Pb-3a3	9.8·10 ⁻⁴	0.35	60	37.1	7.4	8.4	-0.63	-1.0·10 ⁻²
Pb-3b1		0.35	125	9.8	8.0	8.4	-0.66	-5.3·10 ⁻³
Pb-3b2		0.35	150	4.6	8.0	8.4	-0.67	-4.5·10 ⁻³
Pb-3b3		0.35	50	47.6	7.2	8.4	-0.65	-1.3·10 ⁻²
Pb-3b4		0.35	142	3.9	7.8	8.4	-0.69	-4.9·10 ⁻³
Pb-3c1		0.35	141	4.0	7.8	8.4	-0.68	-4.8·10 ⁻³
Pb-3c2		0.35	122	8.0	7.2	8.4	-0.67	-5.5·10 ⁻³
Gv-a1	6.5·10 ⁻⁵	0.12	91	13.4	6.4	2.5	-0.57	-6.3·10 ⁻³
Gv-a2	5.3·10 ⁻⁵	0.12	98	12.2	6.8	2.5	-0.59	-6.0·10 ⁻³
Gv-a3		0.12	86	17.8	6.0	2.5	-0.57	-6.6·10 ⁻³
QS-a1	2.8·10 ⁻⁴	0.16	138	10.4	8.2	2.4	-1.28	-9.3·10 ⁻³
QS-a2		0.16	160	5.8	8.2	2.4	-1.26	-7.9·10 ⁻³
QS-a3	2.5·10 ⁻⁴	0.16	160 [*]	5.0	8.2	2.4	-1.25	-7.8·10 ⁻³
Sd-a1	8.4·10 ⁻⁶	0.14	110	12.8	7.4	3.4	-0.45	-4.1·10 ⁻³
Sd-a2	9.6·10⁻ ⁶	0.14	106	14.1	7.4	3.4	-0.46	-4.3·10 ⁻³
Sd-a3		0.14	109	14.4	7.6	3.4	-0.42	-3.9·10 ⁻³
Sd-a4	7.6·10 ⁻⁶	0.14	113	14.6	7.5	3.4	-0.42	-3.7·10 ⁻³
Sd-b1	7.8·10 ⁻⁶	0.14	152	18.3	7.5	3.4	-0.51	-3.4·10 ⁻³
Sd-b2	8.4·10 ⁻⁶	0.14	147	18.1	7.6	3.4	-0.53	-3.6·10 ⁻³
Sd-b3	5.0·10 ⁻⁶	0.14	151 [*]	16.6	7.6	3.4	-0.48	-3.2·10 ⁻³

5. Modelling the Streaming Potential Phenomenon

Modelling streaming potentials can be a powerful tool in understanding the phenomenon and for interpreting SP anomalies. Many simple models have been produced involving charged bodies of various shapes; however this approach disregards the nature of the sources that generated the potentials. An approach that considers these sources as functions of the existing physical properties and driving forces would thereby provide a more realistic model.

As mentioned in the previous chapter, streaming potentials are the result of electric currents created by the flow of a liquid with respect to a solid. The current flow, defined by Ohm's law and the fluid flow, defined by Darcy's law, can be related one to the other by the theory of coupled flows. This approach was first used in a model by Nourbehecht (1963) drawing on the earlier work of Marshall and Madden (1959). Solutions were formulated in terms of a total- or pseudo-potential.

Although theoretically correct, the pseudo-potential approach misrepresented the actual mechanisms that generate streaming potentials. The pseudo-potential sources occurred at boundaries where there was a change in the streaming potential coefficient; i.e. all along a seepage zone rather than just at the ends where the charge accumulations would occur. The sources were also a result of a pressure; whereas conduction current sources arise due to gradients of pressure according to the actual physical processes involved. Additionally, the pseudo-potential approach is only applicable to hydraulically driven fluid flow and its application to more complex geometries can be difficult.

Coupled flow theory was also used as a starting point by Sill (1983), who developed a new approach for modelling streaming potentials by formulating explicit expressions for the involved current sources. The application of his approach is straight-forward even when complex geometries are involved, and it can be applied equally to non-hydraulically driven fluid flow such as that caused by temperature gradients. Sill's method is more analogous to the actual physical process involved and forms the basis for most modern modelling of the streaming potential phenomenon.

For illustration, a simple model of water flowing through a seepage zone in a dam structure is presented in Figure 5.1. In this case the hydraulic potential difference is the difference between the water levels on each side of the dam (Δh). This potential difference drives liquid to flow through the leakage zone as shown in Figure 5.1a. Using Darcy's law, the flow velocity can be calculated as $v = K\Delta h$, where K is the hydraulic conductivity of the leakage zone material. The liquid flow will shear off and carry with it excess charged particles from the diffuse part of the electric double layer, creating an electric convection current as illustrated in Figure 5.1b.

The convection current density can be expressed as $J_{conv} = C\nabla P$, where *C* is the crosscoupling coefficient of the leakage zone material and ∇P is the pressure gradient. The magnitude of this current density will change at boundaries where either *K* or *C* change, consequently causing an accumulation of charge. In this case the ends of the



Figure 5.1. Schematic representation of liquid flow through a leakage zone in a dam structure.

(a) The height difference between the upstream and downstream water levels (Δ h) is the hydraulic potential difference that drives the fluid flow through the leakage zone. If the leakage zone has a cross-sectional area of **A** and a hydraulic conductivity of **K**, the total flow would be:

Q = **AK**∆h.

(b) The liquid flow carries with it excess charges from the diffuse portion of the electric double layer, creating an electric convection current through the leakage zone. If the leakage zone has a crosscoupling conductivity of **C**, the convection current would be:

$I_{conv} = AC\nabla P$,

where $\nabla \mathbf{P}$ is the pressure gradient.

(c) The density of the convection current changes at boundaries where either **K** or **C** change, resulting in the accumulation of excess charges at these boundaries.

(d) The electric potential difference created by the accumulations of charge in turn drives a conduction current back through the leakage zone, the dam structure and the up and downstream bodies of water, based on their respective electric conductivities.

leakage zone will thus become charged as shown in Figure 5.1c. Negative charges typically form upstream and positive charges downstream, although the opposite can occur under certain conditions. This charge difference constitutes an electric potential difference that in turn drives a conduction current flow back through the structure as illustrated in Figure 5.1d. The exact pattern will be a complex function of the resistivity variations in the structure. The interaction of the conduction current density and this ground resistivity distribution produce the SP anomalies that are measured on the dam surface. Although illustrated in separate stages for simplicity, these processes occur gradually and simultaneously until a steady-state condition is reached.

5.1 Streaming potentials as explicit functions of current sources

The theory of coupled flows introduced in the previous chapter can be used to describe flows of a various type such as liquid, current or heat as a linear function of various driving forces such as hydraulic or electric potential gradients. The relationship between a liquid flow and an electric current flow is represented by equations 4.21 and 4.22:

$$\mathbf{Q} = -K\nabla\xi - C\nabla\phi \quad \text{and} \tag{5.1}$$

$$\mathbf{J} = -C\nabla\xi - \sigma_b\nabla\phi. \tag{5.2}$$

Generally fluid flow does not generate large electrical currents, and conversely, injecting electric current does not generate large fluid flows; therefore the cross-coupling terms in equations 5.1 and 5.2 are typically much smaller than the direct flow terms on the diagonal. Eliminating the cross-coupling terms reduces equations 5.1 and 5.2 to Darcy's law and Ohm's law respectively. Because the effect of a secondary electric current on fluid flow is minimal, the hydraulic potential distribution can in fact be solved for separately using equation 5.1 without the cross-coupling term:

$$\mathbf{Q} = -K\nabla\xi \,. \tag{5.3}$$

Although the effects of electric currents on fluid flow are negligible for most geological materials, care should be taken when modelling materials with very low permeabilities, such as clay. Yeung (1994) suggests that the effects of electric currents on fluid flow can only be safely neglected for soils with hydraulic conductivities exceeding 10^{-9} m/s.

How the hydraulic potential distribution is obtained using the finite element method is discussed below, as well as how the same approach can be used to solve for the electric potential distribution.

Equation 5.2 is now analysed according to the approach developed by Sill (1983). Looking at the equation it is not apparent how and where the streaming potential sources occur; however this can be deduced by expanding the equation. The first term of equation 5.2 represents the convection current driven by the fluid flow and the

second term represents the conduction current driven by the electric potential gradient. Equation 5.2 can thus be rewritten as

$$\mathbf{J}_{total} = \mathbf{J}_{conv} + \mathbf{J}_{cond} .$$
 (5.4)

If conditions are such that no external current sources are imposed and resistivities are not fluctuating (i.e. $\partial \rho / \partial t = 0$), then conservation of charge requires that the total current be divergenceless, i.e. $\nabla \cdot \mathbf{J}_{total} = 0$. When this is the case the conduction and convection currents are related as follows

$$\nabla \mathbf{J}_{cond} = -\nabla \cdot \mathbf{J}_{conv} \quad \text{or} \quad -\nabla (\sigma_b \nabla \phi) = \nabla (C \nabla \xi)$$
 (5.5)

The right-hand side of this equation represents a source term for the conduction current with units of A/m^3 . Expanding this term by applying the divergence theory yields

$$\nabla \cdot \mathbf{J}_{cond} = \nabla C \nabla \xi + C \nabla^2 \xi , \qquad (5.6)$$

which can be further expanded by substituting equation 5.3 into the second term:

$$\nabla \cdot \mathbf{J}_{cond} = \nabla C \nabla \xi - \frac{C}{K} \nabla \cdot \mathbf{Q} + \frac{C}{K^2} \nabla K \cdot \mathbf{Q} .$$
 (5.7)

The three terms in equation 5.7 indicate three possible sources of conduction current:

- 1. The first term shows current sources where there is liquid flow parallel to a gradient of the cross-coupling conductivity, i.e. flow across a boundary where C changes. The source will be positive when the gradients are in the same direction.
- 2. The second term indicates current sources where there are externally imposed sources of liquid flow. The current source will be opposite in sign to the flow source when C > 0, which is the case for most geological materials.
- 3. The third term shows sources where there is fluid or heat flow parallel to a gradient of the hydraulic conductivity, i.e. flow across a boundary where K changes. Secondary sources of fluid flow are induced at these boundaries and when C > 0, the source is positive when the fluid flow (**Q**) is in the same direction as the hydraulic conductivity gradient.

For the simple model presented in Figure 5.1 the conduction current sources occur solely where the liquid flows into and out of the seepage zone; i.e. at the boundaries where the hydraulic and cross-coupling conductivities change. If the dam structure were composed of a porous material then the liquid would flow through the dam body itself and not just through the seepage zone. For this case conduction current sources would exist at all points where the liquid flows in and out of the dam body as well as where liquid flows from the dam body into the leakage zone. A simple schematic of these flows and the generated current sources is presented in Figure 5.2.

The conduction current sources will produce an electrical potential distribution that can be solved for using equation 5.5. Naturally the hydraulic and cross-coupling conductivities can change gradually in a medium rather than abruptly at boundaries. Should this be the case then a volume charge density would accumulate rather than a surface charge density. The finite element method introduces boundaries by subdividing the flow areas into smaller, homogeneous elements and assigning them different conductivity values. Gradual changes of the conductivities can be approximated by using a finer element mesh.



Figure 5.2. Schematic representation of liquid flow through a dam structure containing a leakage zone. The dam structure above the water table (phreatic-surface) together with the air and the water outside of the dam structure can each have assumed hydraulic and crosscoupling conductivity values of zero. If the dam structure has hydraulic conductivity K_1 and cross-coupling conductivity C_1 , and the seepage zone has hydraulic and crosscoupling conductivity values of K_2 and C_2 , respectively, where $K_2 > K_1$ and $C_2 > C_1$, then (a) flow across the boundaries where values of K and C change will result in (b) the accumulation of excess charges on those boundaries. These charge accumulations constitute the sources of the conduction current.

5.2 Numerical Modelling

Using numerical methods to model the streaming potential phenomenon can be seen as a three step procedure. The first step is to determine the hydraulic potential distribution (ζ), generally by solving a variant of equation 5.3. The second step involves using the obtained hydraulic potential distribution to solve the right-hand side of equation 5.5 for the location and magnitude of the conduction current sources. The final step of the procedure is using the obtained conduction current sources to solve equation 5.5 for the electric potential distribution (ϕ).

5.2.1 Pressure distribution

In order to determine the hydraulic potential distribution in a region, Darcy's law (equation 5.3) is combined with the law that states that pressure is conserved within the region if there are no external sources, i.e. $\nabla \cdot \mathbf{Q} = 0$. These two equations combine to produce

$$\nabla (K \nabla \xi) = 0, \qquad (5.8)$$

which has the same form as either term of equation 5.5. This means that the following procedure can be applied to all three steps in obtaining the electric potential distribution.

In three dimensions the Cartesian form of equation 5.8 can be generalised as

$$\frac{\partial}{\partial x} \left(K_x \frac{\partial \xi}{\partial x} \right) + \frac{\partial}{\partial y} \left(K_y \frac{\partial \xi}{\partial y} \right) + \frac{\partial}{\partial z} \left(K_z \frac{\partial \xi}{\partial z} \right) + q = 0, \qquad (5.9)$$

where q represents any external sources. This equation, also known as Poisson's equation or the quasi-harmonic equation, describes the steady state behaviour of many physical phenomena in a non-homogeneous and anisotropic region (with axes of anisotropy parallel to the co-ordinate axes). When solving practical problems of groundwater flow, the hydraulic conductivity is not usually expressed as a continuous function of the co-ordinates. Instead, the region is divided into a number of zones characterised by different, yet within the zone constant values of hydraulic conductivity. This simplification is well suited to the finite element method, as the region is divided into elements which can be assigned different material properties.

In order to solve a groundwater flow problem it is necessary to know not only the differential equation but also the boundary conditions on the domain boundary. Two types of boundary condition can occur:

1. what is termed an essential boundary condition, where a pressure value is prescribed at points on the boundary:

$$\xi = \xi_0 \,, \tag{5.10}$$

where ξ_0 is the prescribed pressure value;

2. and a natural boundary condition, which describes a 'loading' of the boundary surface by prescribing a value for the hydraulic potential derivative, i.e. a flow rate (q_0) in the outward direction normal to the boundary such that

$$K_{x}\frac{\partial\xi}{\partial x}n_{x} + K_{y}\frac{\partial\xi}{\partial y}n_{y} + K_{z}\frac{\partial\xi}{\partial z}n_{z} = -q_{0}, \qquad (5.11)$$

where n_x , n_y and n_z are the direction cosines between the outward normal (*n*) and the *x*, *y* and *z* axes respectively. If $q_0 = 0$ and the domain were isotropic $K_x = K_y = K_z = K$, then equation 5.11 would reduce to $K \partial \xi / \partial n = 0$. This would indicate that the pressure gradient in the direction normal to the surface is zero; therefore this portion of the surface would be impermeable.

At this point the Galerkin weighted residual method (Kazda, 1990) will be used to reduce the problem to a finite set of equations and unknowns. The Galerkin method is a common numerical method found in most numerical modelling texts. First, rather than zero, equations 5.9 and 5.11 are set equal to e_V and e_S respectively, which are the errors involved in satisfying these equations by the discretisation approximation. e_V is the error in satisfying equation 5.9 over the volume of the region and e_S is the error in satisfying any natural boundary conditions on the surface. An essential boundary condition is exactly satisfied by prescribing values for ξ and there is no error involved. Weighted residual approaches such as the Galerkin method then attempt to minimise the residuals e_V and e_S by using suitably chosen weighting functions w_V and w_S . Specifically, it is required that

$$\int_{V} e_{V} w_{V} dV + \int_{S_{N}} e_{S} w_{S} dS = 0, \qquad (5.12)$$

where V denotes the volume of the domain and S_N denotes the part of the boundary on which natural boundary conditions apply. Substituting equations 5.9 and 5.11 yields

$$\int_{V} \left[\frac{\partial}{\partial x} \left(K_{x} \frac{\partial \xi}{\partial x} \right) + \frac{\partial}{\partial y} \left(K_{y} \frac{\partial \xi}{\partial y} \right) + \frac{\partial}{\partial z} \left(K_{z} \frac{\partial \xi}{\partial z} \right) + q \right] w_{V} dV + \int_{S_{N}} \left[K_{x} \frac{\partial \xi}{\partial x} n_{x} + K_{y} \frac{\partial \xi}{\partial y} n_{y} + K_{z} \frac{\partial \xi}{\partial z} n_{z} + q_{0} \right] w_{S} dS = 0.$$
(5.13)

Two functions, a and b, that are continuous including their partial derivatives in a closed domain can be related by Green's theorem (Hildebrand, 1965) as:

$$\int_{V} \frac{\partial a}{\partial \omega} b dV = \int_{S} a b n_{\omega} dS - \int_{V} a \frac{\partial b}{\partial \omega} dV, \qquad (5.14)$$

where ω is either *x*, *y* or *z*. Applying Green's theorem to the second order derivatives in the first integral of equation (5.13) gives

$$\int_{S} \left[K_{x} \frac{\partial \xi}{\partial x} n_{x} + K_{y} \frac{\partial \xi}{\partial y} n_{y} + K_{z} \frac{\partial \xi}{\partial z} n_{z} \right] w_{V} dS$$

$$- \int_{V} \left[K_{x} \frac{\partial \xi}{\partial x} \frac{\partial w_{V}}{\partial x} + K_{y} \frac{\partial \xi}{\partial y} \frac{\partial w_{V}}{\partial y} + K_{z} \frac{\partial \xi}{\partial z} \frac{\partial w_{V}}{\partial z} - q w_{V} \right] dV$$

$$+ \int_{S_{N}} \left[K_{x} \frac{\partial \xi}{\partial x} n_{x} + K_{y} \frac{\partial \xi}{\partial y} n_{y} + K_{z} \frac{\partial \xi}{\partial z} n_{z} + q_{0} \right] w_{S} dS = 0.$$
(5.15)

If the weighing functions are chosen so that $w_S = -w_V$ then the first and third integrals of equation 5.15 can be summed, resulting in

$$\int_{S_{E}} \left[K_{x} \frac{\partial \xi}{\partial x} n_{x} + K_{y} \frac{\partial \xi}{\partial y} n_{y} + K_{z} \frac{\partial \xi}{\partial z} n_{z} \right] w dS - \int_{S_{N}} q_{0} w dS$$
$$- \int_{V} \left[K_{x} \frac{\partial \xi}{\partial x} \frac{\partial w}{\partial x} + K_{y} \frac{\partial \xi}{\partial y} \frac{\partial w}{\partial y} + K_{z} \frac{\partial \xi}{\partial z} \frac{\partial w}{\partial z} - qw \right] dV = 0, \qquad (5.16)$$

where S_E is the part of the boundary on which essential boundary conditions apply. At this point it is necessary to select points in the region for which the hydraulic potential will be calculated, thereby obtaining the hydraulic potential distribution. This was achieved by using the finite element method.

5.2.2 Finite element discretisation

The finite element method (FEM) approximates the continuous unknown function (ξ) by a discrete model which consists of a set of values for the function at a finite number of pre-selected points in the domain, together with piecewise approximations of the function over a finite number of disjunct sub domains. These sub domains are the finite elements and are formed from the pre-selected points, which are called nodes.

Finite element discretisation is achieved by approximating the unknown function as

$$\xi = \sum_{j=1}^{n} N_{j} \,\xi_{j} \,, \tag{5.17}$$

where *n* is the total number of nodes, ξ_j is the value of ξ at node *j* and N_j is the global shape function of node *j*. Shape functions are interpolating polynomials that define the unknown function over each element. The shape function of each node is calculated from the type and geometry of each element that it is in contact with. Shape functions are discussed further below. For the Galerkin method, the weighting function (*w_i*) corresponding to each node *i* is conveniently chosen so that $w_i = N_i$. Substituting equation 5.17 into 5.16 and setting $w_i = N_i$ yields *n* equations of the form

$$\int_{S_{E}} N_{i} \sum_{j=1}^{n} \left[K_{x} \frac{\partial N_{j}}{\partial x} n_{x} + K_{y} \frac{\partial N_{j}}{\partial y} n_{y} + K_{z} \frac{\partial N_{j}}{\partial z} n_{z} \right] \xi_{j} dS - \int_{S_{N}} q_{0} N_{i} dS$$
$$- \int_{V} \left[\sum_{j=1}^{n} \left[K_{x} \frac{\partial N_{j}}{\partial x} \frac{\partial N_{i}}{\partial x} + K_{y} \frac{\partial N_{j}}{\partial y} \frac{\partial N_{i}}{\partial y} + K_{z} \frac{\partial N_{j}}{\partial z} \frac{\partial N_{i}}{\partial z} \right] \xi_{j} - q N_{i} \right] dV = 0. \quad (5.18)$$

From equation 5.16 it is clear that the first integral of equation 5.18 represents a flow rate of water through nodes on the boundary where an essential boundary condition applies. However, according to equation 5.10 the pressure is prescribed at these nodes (ξ_0) . The integral therefore represents the 'reaction' associated with the prescribed value and can be disregarded here. After rearranging and changing the order of summation and integration, equation 5.18 now takes the form of

$$\sum_{j=1}^{n} \int_{V} \left[K_{x} \frac{\partial N_{i}}{\partial x} \frac{\partial N_{j}}{\partial x} + K_{y} \frac{\partial N_{i}}{\partial y} \frac{\partial N_{j}}{\partial y} + K_{z} \frac{\partial N_{i}}{\partial z} \frac{\partial N_{j}}{\partial z} \right] dV \cdot \xi_{j}$$

$$= \int_{V} q N_{i} dV - \int_{S_{N}} q_{0} N_{i} dS . \qquad (5.19)$$

The *n* equations in the form of equation 5.19 can be expressed in matrix form as $\mathbf{K} \cdot \boldsymbol{\xi} = \mathbf{f}$, where **K** is an $n \times n$ 'stiffness' matrix of the form

$$k_{ij} = \int_{V} \left[K_x \frac{\partial N_i}{\partial x} \frac{\partial N_j}{\partial x} + K_y \frac{\partial N_i}{\partial y} \frac{\partial N_j}{\partial y} + K_z \frac{\partial N_i}{\partial z} \frac{\partial N_j}{\partial z} \right] dV, \qquad (5.20)$$

f is a size *n* 'force' vector and ξ is the size *n* unknown vector. This formulation represents *n* equations and *n* unknowns, allowing for the solution of the hydraulic potential at each node. The force vector will only be nonzero in the equations that correspond to nodes where there is a natural boundary condition (q_0) and nodes corresponding to a point source or sink (q). An essential boundary condition simplifies the solution by reducing the number of unknowns. Each equation corresponding to a node with an essential boundary condition can be eliminated while the prescribed pressure value (ξ_0) is incorporated into the remaining equations. When all boundary conditions are accounted for, the resulting system of equations can be solved, providing the hydraulic potential distribution in the form of a pressure value at each node.

5.2.3 Electrical potential distribution

A similar approach can be used for obtaining the electric potential distribution. Equation 5.5 represents a Poisson type equation similar in form to equation 5.3 that can be generalised in three-dimensional Cartesian form as

$$\frac{\partial}{\partial x} \left(\sigma_x \frac{\partial \phi}{\partial x} \right) + \frac{\partial}{\partial y} \left(\sigma_y \frac{\partial \phi}{\partial y} \right) + \frac{\partial}{\partial z} \left(\sigma_z \frac{\partial \phi}{\partial z} \right) + \nabla \cdot \left(C \nabla \xi \right) + v = 0, \quad (5.21)$$

where v represents any external current sources and $\nabla \cdot (C\nabla \zeta)$ represents the conduction current sources previously discussed. Similar essential and natural boundary conditions can also be described for the region as follows

$$\phi = \phi_0, \text{ and} \tag{5.22}$$

$$\sigma_x \frac{\partial \phi}{\partial x} n_x + \sigma_y \frac{\partial \phi}{\partial y} n_y + \sigma_z \frac{\partial \phi}{\partial z} n_z = -v_0, \qquad (5.23)$$

where ϕ_0 is a prescribed electric potential value and v_0 is a prescribed value for the electric potential derivative in the outward direction normal to the boundary.

Using the Galerkin weighted residual method in the same manner as for the hydraulic potential distribution; the electric potential problem can be reduced to a set of n equations and unknowns of the form:

$$\sum_{j=1}^{n} \int_{V} \left[\sigma_{x} \frac{\partial N_{i}}{\partial x} \frac{\partial N_{j}}{\partial x} + \sigma_{y} \frac{\partial N_{i}}{\partial y} \frac{\partial N_{j}}{\partial y} + \sigma_{z} \frac{\partial N_{i}}{\partial z} \frac{\partial N_{j}}{\partial z} \right] dV \cdot \phi_{j}$$

$$= \int_{V} \left[\nabla \cdot (C \nabla \xi) + v \right] N_{i} dV - \int_{S_{N}} v_{0} N_{i} dS, \qquad (5.24)$$

where *n* is again the total number of nodes used in the finite element discretisation. This equation, $\mathbf{\sigma} \cdot \mathbf{\phi} = \mathbf{f}$ in matrix form, is very similar to equation 5.19, the only differences being the electric conductivity replacing the hydraulic conductivity in the stiffness matrix and the addition of the conduction current source term in the force vector. This term can be easily calculated for each node and input into the force vector when the same element discretisation is used for both the hydraulic and electric potential problems.

The conduction current source term for each equation can be expressed as

$$\int_{V} \nabla \cdot (C \nabla \xi) N_{i} dV = \sum_{j=1}^{n} \int_{V} \left[C_{x} \frac{\partial N_{i}}{\partial x} \frac{\partial N_{j}}{\partial x} + C_{y} \frac{\partial N_{i}}{\partial y} \frac{\partial N_{j}}{\partial y} + C_{z} \frac{\partial N_{i}}{\partial z} \frac{\partial N_{j}}{\partial z} \right] dV \cdot \xi_{j}, \quad (5.25)$$

which can also be expressed in matrix form as $C \cdot \xi$. Because the hydraulic potential has already been solved for at each node, simply formulating the cross-coupling conductivity stiffness matrix and multiplying it by the hydraulic potential vector results in the conduction current source value for each node. When this is done and any other boundary conditions are accounted for, the system of equations can be solved for the electric potential distribution.

5.2.4 Element types and shape functions

The accuracy of the results obtained by the finite element method depends heavily on the type of finite element applied. Five different element types were used in the development of the computer programs written by the author for this project. The program first defines the geometry of the region and solves for the hydraulic potential distribution based on defined hydraulic conditions. The pressure distribution thus obtained is then used with defined cross-coupling conditions to calculate the conduction current source terms. Finally, these source terms are used with defined electric conductivity conditions to solve for the electric potential distribution. The same element discretisation is used for each stage of the solution in order to simplify the solutions.

The program initially used two-dimensional linear triangular elements with three nodes due to their simplicity and versatility for modelling complex two-dimensional shapes. Triangular elements retain their simplicity in three dimensions, requiring only a single additional node to form a four-sided tetrahedron; however it is actually very difficult to divide even a simple three-dimensional region into elements of this form.

For this reason the program was re-written to use quadrilateral elements, also known as isoparametric elements.

Quadrilateral elements are nearly as convenient as triangular elements for subdividing a region; however defining the interpolating shape functions for a quadrilateral shape is not as straight-forward as for a triangle. For this reason a transformation is commonly used by which each element defined in the global coordinate system (x, y)is mapped onto a square with a side length of 2 units and its centre located at the origin of a local coordinate system (r, s). The shape functions are evaluated for each node on the simple square and then mapped to the correspond nodes in the global coordinate system. The procedure for doing this is analogous to that of interpolating the unknown function (ξ) in equation 5.17. If the element has k nodes, then the following relationships are true:

$$x = \sum_{i=1}^{k} N_i(r, s) x_i$$
 and (5.26)

$$y = \sum_{i=1}^{k} N_i(r, s) y_i .$$
 (5.27)

The global coordinates are thus obtained by substituting the local coordinate shape functions into equations 5.26 and 5.27. In practice it is the partial derivatives of the shape functions with respect to the global coordinates that are required, as seen in equations 5.19, 5.24 and 5.25. These can be obtained through the following equations:

$$\frac{\partial N_i}{\partial r} = \frac{\partial N_i}{\partial x} \frac{\partial x}{\partial r} + \frac{\partial N_i}{\partial y} \frac{\partial y}{\partial r} \quad \text{and}$$
(5.28)

$$\frac{\partial N_i}{\partial s} = \frac{\partial N_i}{\partial x} \frac{\partial x}{\partial s} + \frac{\partial N_i}{\partial y} \frac{\partial y}{\partial s}, \qquad (5.28)$$

which can be written in matrix form as:

$$\begin{bmatrix} \frac{\partial N_i}{\partial r} \\ \frac{\partial N_i}{\partial s} \end{bmatrix} = \begin{bmatrix} \frac{\partial x}{\partial r} & \frac{\partial y}{\partial r} \\ \frac{\partial x}{\partial s} & \frac{\partial x}{\partial s} \end{bmatrix} \cdot \begin{bmatrix} \frac{\partial N_i}{\partial x} \\ \frac{\partial N_i}{\partial y} \end{bmatrix} = \mathbf{J} \cdot \begin{bmatrix} \frac{\partial N_i}{\partial x} \\ \frac{\partial N_i}{\partial y} \end{bmatrix},$$
(5.29)

where J is known as the Jacobian matrix and its determinant, called the Jacobian, plays a significant role in the uniqueness of the mapping between the local and global coordinate systems.

From equation 5.29 it becomes clear that the partial derivatives of the shape functions with respect to the global coordinates are obtained using the inverse of the Jacobian matrix (J^{-1}) , should it exist. In order for its inverse to exist, the Jacobian matrix must be regular, i.e. its determinant, the Jacobian, must be non-zero. It can be shown that the Jacobian is greater than zero at all points of a quadrilateral element when all of its

interior corner angles are less than 180°. This should be kept in mind when using unusually shaped elements.

The first type of quadrilateral element used was a four-sided 2-D element with eight nodes, located one at each corner and one midway along each side as shown in Figure 5.3a. The side nodes allow the edges of the element to take the form of second degree parabolic arcs, thus earning elements with three nodes per side the name parabolic elements. If the nodes are numbered as shown in Figure 5.3a, then the derived shape functions for the local coordinate system are:

$$N_{1} = \frac{1}{4} (1+r)(1+s) - \frac{1}{2}N_{8} - \frac{1}{2}N_{5}$$
(5.30)

$$N_2 = \frac{1}{4} (1 - r)(1 + s) - \frac{1}{2} N_5 - \frac{1}{2} N_6$$
(5.31)

$$N_{3} = \frac{1}{4} (1 - r)(1 - s) - \frac{1}{2} N_{6} - \frac{1}{2} N_{7}$$
(5.32)

$$N_4 = \frac{1}{4} (1+r)(1-s) - \frac{1}{2}N_7 - \frac{1}{2}N_8$$
(5.33)

$$N_5 = \frac{1}{2} \left(1 - r^2 \right) \left(1 + s \right) \tag{5.34}$$

$$N_6 = \frac{1}{2} (1 - r) (1 - s^2)$$
(5.35)

$$N_7 = \frac{1}{2} (1 - r^2) (1 - s)$$
(5.36)

$$N_8 = \frac{1}{2} (1+r) (1-s^2)$$
(5.37)

Obtaining the partial derivative of the shape functions with respect to the local coordinate system required for equation 5.29 is straightforward. Although this element type has more nodes, and thus more equations than linear element types, its bi-quadratic shape functions provide much more accurate results, even when the domain is divided into a far greater number of linear elements.

The program was then written to incorporate a three-dimensional version of this parabolic element. Each element is a hexahedron with twenty nodes, one at each of the eight corners and one midway between each adjacent corner, as shown in Figure 5.3b. The exact same procedure used for two-dimensional finite elements can be applied to three-dimensional elements. Equation 5.29 would thus become

$$\begin{bmatrix} \frac{\partial N_i}{\partial r} \\ \frac{\partial N_i}{\partial s} \\ \frac{\partial N_i}{\partial t} \end{bmatrix} = \begin{bmatrix} \frac{\partial x}{\partial r} & \frac{\partial y}{\partial r} & \frac{\partial z}{\partial r} \\ \frac{\partial x}{\partial s} & \frac{\partial y}{\partial s} & \frac{\partial z}{\partial s} \\ \frac{\partial x}{\partial t} & \frac{\partial y}{\partial t} & \frac{\partial z}{\partial t} \end{bmatrix} \cdot \begin{bmatrix} \frac{\partial N_i}{\partial x} \\ \frac{\partial N_i}{\partial y} \\ \frac{\partial N_i}{\partial z} \end{bmatrix} = \mathbf{J} \cdot \begin{bmatrix} \frac{\partial N_i}{\partial x} \\ \frac{\partial N_i}{\partial y} \\ \frac{\partial N_i}{\partial z} \end{bmatrix},$$
(5.38)

where *t* is the third dimension in the local coordinate system corresponding to *z*.



Figure 5.3. Two- and three-dimensional parabolic isoparametric elements with eight and twenty nodes, respectively, in the global and the local system of coordinates. Two- and three dimensional linear isoparametric elements with four and eight nodes, respectively, are obtained by removing the mid-side nodes from these elements.

The local shape functions for the eight corner nodes of the element shown in Figure 5.3b have the form

$$N_{i} = \frac{1}{8} (1 + r_{i}r)(1 + s_{i}s)(1 + t_{i}t) - \frac{1}{2} (N_{a} + N_{b} + N_{c}), \qquad (5.39)$$

where r_i , s_i , t_i are the local coordinates (1, 0 or -1) of node i = 1 to 8 and N_a , N_b , N_c are the local shape functions of the three mid-side nodes adjacent to node i. The remaining twelve local shape functions for the mid-side nodes have a simpler form:

$$N_{9} = \frac{1}{4} \left(1 - r^{2} \right) \left(1 + s \right) \left(1 + t \right)$$
(5.40)

$$N_{10} = \frac{1}{4} (1 - r) (1 - s^2) (1 + t)$$
(5.41)

$$N_{11} = \frac{1}{4} \left(1 - r^2 \right) \left(1 - s \right) \left(1 + t \right)$$
(5.42)

$$N_{12} = \frac{1}{4} (1+r)(1-s^2)(1+t)$$
(5.43)

$$N_{13} = \frac{1}{4} (1 - r^2) (1 + s) (1 - t)$$
(5.44)

$$N_{14} = \frac{1}{4} (1 - r) (1 - s^2) (1 - t)$$
(5.45)

$$N_{15} = \frac{1}{4} (1 - r^2) (1 - s) (1 - t)$$
(5.46)

$$N_{16} = \frac{1}{4} (1+r)(1-s^2)(1-t)$$
(5.47)

$$N_{17} = \frac{1}{4} (1+r)(1+s)(1-t^2)$$
(5.48)

$$N_{18} = \frac{1}{4} (1 - r)(1 + s)(1 - t^2)$$
(5.49)

$$N_{19} = \frac{1}{4} (1 - r)(1 - s)(1 - t^2)$$
(5.50)

$$N_{20} = \frac{1}{4} (1+r)(1-s)(1-t^2)$$
(5.51)

For three-dimensional problems it was found that having twenty nodes per element became quite taxing for even powerful computers as more elements were used to subdivide a region. For this reason the option to use linear quadratic elements was included in the program. Both two- and three-dimensional linear elements were incorporated and these appear similar to the elements presented in Figure 5.3, only without the nodes at the midpoint of each side. The shape functions for these elements are equal to the corner node shape functions of their parabolic equivalents (equations 5.30-33 and 5.39) without the mid-side node terms. By using linear elements the model can be subdivided into a larger number of elements without overloading a computers memory and processor.

All of the elements used employ Lagrangian interpolation, which means that only the values of the unknown function at the nodes are used. This allows relatively easy derivation of the shape functions. An element with four nodes per side would have a cubic shape function, and successively adding more nodes further increases the degree of the interpolating polynomial. An increase in the number of nodes however increases the matrix sizes and the resulting equation system, requiring more data entry and computer processing. Nevertheless Hermitian interpolation, which uses derivative values as well as function values of the unknown function, can be applied on elements with at least a cubic interpolating polynomial. This was not attempted for this thesis due to the many difficulties involved with Hermitian interpolation, among them the inherent increased data processing, the complexity of Hermitian element formulation and the difficulties in dividing a domain of common shape into elements of this type.

6. SPiso3D: a finite element program for modelling streaming potentials

The program SPiso3D has evolved from a purely text-based input/output twodimensional modelling DOS program to a fully graphical three-dimensional modelling program. The first incarnation of the program used triangular linear elements and was called SPtri2D. In order to incorporate the third dimension without losing the accuracy of triangular elements, the switch was made to parabolic isoparametric elements and the program was renamed SPiso3D. Later it was found that linear isoparametric elements didn't greatly affect the accuracy of results but reduced the data processing requirements and thus the program was rewritten to include these elements as well. Details concerning these element types are provided in the preceding chapter.

The program was written in the Fortran 90 programming language using Compaq Visual Fortran version 6.6C and can be run as a stand-alone program on any modern personal computer (PC). The on-screen images are drawn using the OpenGL graphics application program interface (API) and some aspects of their appearance will vary from machine to machine based on the installed graphics card and video drivers. Although the final program has only been tested on Microsoft Windows operating systems, it should also work with other operating systems.

6.1 Checking the results

In the early stages of programming, efforts were made to verify the results of the program. Comparisons were made with analytical models as well as an existing twodimensional finite difference modelling program. The results are discussed in this section.

6.1.1 Analytical solution

Equation 5.3 can be rewritten into the more familiar form of Darcy's law as follows:

$$Q = -\frac{kA}{\mu} \nabla P \,, \tag{6.1}$$

where Q is the flow rate, k is permeability, A is the domain's cross-sectional area, μ is the dynamic viscosity of the fluid and P is the pressure. Permeability is related to the hydraulic conductivity as $K = k \cdot g \sigma / \mu$, where g is the acceleration of gravity and σ is the fluid density. Permeability is expressed in Darcy units after H. Darcy who first studied the flow of liquids through porous media in the 19th century. Specifically one Darcy is equal to 0.987 × 10⁻¹² m².

For a single point source in a homogeneous earth, equation 6.1 can be solved for the pressure to give

$$P(r) = -\frac{Q\mu}{4\pi rk},\tag{6.2}$$

where r is the distance from the source to the observation point. For a homogeneous half-space, the pressure from the image source must also be calculated, yielding

$$P(r,r') = -\frac{Q\mu}{4\pi k} \left(\frac{1}{r} + \frac{1}{r'}\right).$$
 (6.3)

Recalling that $\mathbf{J}_{Total} = 0$ within a homogeneous domain, it follows from equation 5.2 that the electrical potential will have the form

$$\phi(r,r') = -\frac{C}{\sigma} \cdot \frac{Q\mu}{4\pi k} \left(\frac{1}{r} + \frac{1}{r'}\right). \tag{6.4}$$

Substituting the streaming potential coefficient S from equation 4.23 into this equation yields

$$\phi(r,r') = \frac{SQ\mu}{4\pi k} \left(\frac{1}{r} + \frac{1}{r'}\right).$$
(6.5)

From this equation a Fortran program called SPanalyt was written which could calculate a two-dimensional potential distribution for any number of point sources at any three-dimensional position within a homogeneous half-space.

Unfortunately this solution could only be applied to a homogeneous half-space, and such an infinite domain is not easily modelled with the finite element method. One solution to the problem of infinite domains was to implement infinite elements (Bettess, 1977; 1980); however these had several disadvantages including complex code, complicated input data files and difficulties in choosing parameters for optimal accuracy.

Another method that avoided the disadvantages of infinite elements was the use of quasi-infinite elements (Kazda, 1990). These are simply elements which are extraordinarily elongated in the required direction, i.e. in the given direction they have a length corresponding to at least one half of the remaining domain part and their other dimension is at least ten times smaller. The problem with modelling a half-space with this type of element was that the corner elements would be elongated in more than one direction and will therefore not be true quasi-infinite elements. Despite this, quasi-infinite elements were used to model a homogeneous half-space containing one point pressure source and one point pressure sink, and the results were compared with those of the analytical solution program.

Figure 6.1a shows a model of a homogeneous half-space containing two point pressure sources. The pressure distribution calculated by SPanalyt using equation 6.3 is presented in Figure 6.1b. It is clear from equation 6.5 that for a homogeneous half-space the electrical potential at any point is simply the product of the pressure at that



Figure 6.1. (a): model section through a homogeneous half-space containing one point pressure sink (left) and one point pressure source (right). (b): the hydraulic pressure distribution (kPa) calculated by the program SPanalyt. (c): the electric potential distribution (mV) calculated by the program SPiso3D. (d): the electric potential distribution (mV) calculated by the program SPPC. The slight distortions in the contour lines of the SPPC solution are interpolation artefacts due to the larger spacing between solution points.

point and the streaming potential coefficient S. Multiplying the values of the contour lines in Figure 6.1b by S (-0.5 mV/kPa) would therefore produce the electrical potential distribution.

Figure 6.1c displays the potential distribution calculated by SPiso3D, which can be seen to match the calculated analytical solution. The large corner elements (not shown) introduced by using quasi-infinite elements in two dimensions did not adversely affect the solution, which satisfactorily matches the analytical solution. The grid points shown in each figure represent the positions of the element nodes for the models used. The programs calculate solution values for each of these points.

6.1.2 SPPC solution

The Fortran program SPPC (Wilt and Butler, 1990) is a finite difference program for calculating SP anomalies due to thermal and pressure sources. The code is a version of program SPXCPL (Sill and Killpack, 1982; Sill 1983) that has been modified for use on a personal computer running MSDOS. The program allows the user to assign different values of permeability, cross-coupling coefficient and resistivity to 52×11 cells (length × depth) of an inhomogeneous half-space, and permits up to nine point or line sources.

Using the same homogeneous model with a point sink and a point source, the SPPC program produced the potential distribution shown in Figure 6.1d, which is seen to match both the analytical and SPiso3D solutions. The slight distortions of the contour lines were caused by the wide spacing of the output data points. The interpolating program thus had less data from which to plot the contour lines. The program only provided output data for the grid points shown, although it used many more nodes to obtain the solution.

Figure 6.2a shows a half-space model similar to the previous, only with a vertical contact introduced. The hydraulic conductivity to the right of the contact was decreased by a factor of two. The resulting electric potential distributions calculated by SPiso3D and SPPC are presented in Figures 6.2b and 6.2c, respectively. Both figures display distributions matching the homogeneous solution to the left of the contact and a doubling of this distribution to the right, which of course makes sense. Leaving the permeability homogeneous but increasing the streaming potential coefficient to the right of the contact by a factor of two would naturally result in the same potential distribution. The contour lines toward the bottom of the model can be seen to differ slightly between the two program results. This is most likely an effect of the quasi-infinite elements used for the SPiso3D model.

The program SPiso3D also outputs the values of the force vector used in the solution, i.e. the conduction current source terms calculated from equation 5.24. When plotted, as shown in Figure 6.2d, it is apparent that the conduction source terms are non-zero at the locations of the source and sink as well as at the boundary where the hydraulic conductivity changes. This confirms two of the three possible sources of conduction current discussed in the previous chapter. The current sources are also opposite in sign to the flow sources, which is expected for the case of C > 0. The zigzag shapes of the contours are interpolation anomalies caused by the plotting program.



Figure 6.2. (a): model section through a half-space containing a point pressure sink and a point pressure source as well as a vertical contact where the hydraulic conductivity changes by a factor of two. (b): the electric potential distribution (mV) calculated by SPiso3D. (c): the electric potential distribution (mV) calculated by SPPC. (d): plot of the non-zero force vector terms calculated by SPiso3D indicating the locations of conduction current sources. The wavy and slightly distorted contour lines are interpolation artefacts caused by irregular node spacing.

A third model of a half-space consisting of a low resistivity layer overlying a high resistivity foundation is presented in Figure 6.3a. The streaming potential coefficient also changes at the layer boundary as well as at a vertical contact in the low resistivity layer. The hydraulic conductivity distribution was left homogeneous for this model.

The electric potential distributions calculated by SPiso3D and SPPC are presented in Figures 6.3b and 6.3c, respectively. Although still very similar, the two solutions do display some slight differences. The program SPiso3D showed slightly narrower contour intervals outside of the point sources as well as a flattening out of the contours as they approach the edges of the model. This 'squashing' of the contour lines as they approach the edges is very likely caused by the use of the quasi-infinite elements to simulate an infinite domain. Besides this effect at the edges of the model, the two solutions show very similar potential distributions.

A plot of the non-zero force vector terms calculated by SPiso3D is presented in Figure 6.3d, indicating the locations of conduction current sources. These are observed to occur at the boundaries where the streaming potential coefficient changes, confirming the remaining possible source of conduction current discussed in the previous chapter, i.e. flow across a boundary where C changes. Note that all of the current sources were limited exclusively to the boundary and flow source nodes; however the contours were plotted between these nodes and those adjacent, producing the odd diamond shapes.

6.2 Program layout

A brief overview of SPiso3D's user interface is presented in Figure 6.4. The program starts up in full-screen but can be run windowed and resized as desired. The window's title bar will display the name of the model once it has been saved to a file. The menu bar contains five submenus: *File*, *Display*, *View*, *Model* and *Help*.

The *File* submenu contains commands to load and save model files as well as the option to exit the program. Models are saved as text files with an *.sp2* extension. These files can be edited manually if desired by any text editor, although it is best to modify them through the program. The files contain the model geometry, the location and magnitude of any boundary conditions as well as the hydraulic, electric and cross-coupling conductivities of each element. The hydraulic and electric solutions are saved to a separate text file of the same name but with a *.dat* extension. If the model is not saved before obtaining a solution then the solution is saved to *unnamed.dat*. These files contain the coordinates, hydraulic and electric potential values and the force vector value from equation 5.19 or 5.24 for each node. The solution files can be imported as spreadsheets into external programs for further interpolation.

The *Display* submenu contains a selection of advanced options for viewing the model. Lines, points and surfaces can be viewed as opaque or slightly transparent in order to keep track of underlying layers. Three-dimensional models can be viewed either in perspective or plane orthographic mode. With perspective, items of the same size will appear smaller with increasing distance; whereas in orthographic mode distance will not affect their size. Three-dimensional objects can also be rotated about various axis.



Figure 6.3. (a): model section through a half-space with a point pressure sink and a point pressure source as well as areas of different electric and cross-coupling conductivities. (b): the electric potential distribution (mV) calculated by SPiso3D. (c): the electric potential distribution (mV) calculated by SPPC. (d): plot of the non-zero force vector terms calculated by SPiso3D indicating the locations of conduction current sources. The wavy and slightly distorted contour lines are interpolation artefacts caused by irregular node spacing.

The model can be viewed so that it fills the length and height of the window or an aspect ration of 1:1 can be selected. How nodes are drawn will vary from computer to computer, thus an option to adjust their size has been included. The user can also choose to zoom in on any portion of the model.

The *View* submenu allows the user to select which aspect of the model is displayed onscreen. There are eight options to choose from: the hydraulic, electric and cross-coupling conductivity distributions, the hydraulic and electric driving forces, the hydraulic potential distribution as pressure head (m) or pressure (kPa), and the electric potential distribution. Obviously the hydraulic and electric potential distributions will display the elements in colours that correspond to their assigned conductivity values. Similarly, selecting the driving forces will display the nodes in colours that correspond to assigned boundary condition values. The hydraulic and electric potential distributions are represented by colour-contoured elements that correspond to a colour scale that appears along the right edge of the window.

The *Model* submenu contains the key commands for defining, modifying and solving a model. Here the user can define a new model by selecting the number of elements in each direction and the type of element used. A model's geometry can be modified by defining the corner coordinates as well as the spacings in each direction. The coordinates of individual nodes can also be adjusted manually using the mouse, allowing more complex geometries to be modelled. The *Conductivities* submenu item lets the user define the various conductivity values and assign them to individual elements. The *Driving forces* submenu item works similarly to define boundary conditions and apply them to nodes. This submenu also contains the commands for solving the hydraulic and electric potential problems. The commands in the *Model* submenu are discussed in greater detail in the following section.

The *Help* submenu contains information concerning the program as well as access to the program's help files.

Located below the menu bar is a toolbar with buttons that correspond to a number of the more frequently used menu commands. These include creating a new model, file access, zooming, the eight viewing options and adjusting the conductivities and driving forces. At the end of the toolbar is the slice dialogue containing an integer value and three buttons corresponding to the xy, yz and zx planes. This dialogue is only active for three-dimensional models. Because it can be quite confusing to look at all of the elements or nodes of a three-dimensional model simultaneously, only one layer of the selected plane is displayed on-screen while the remaining elements are shown only as faint outlines. Which layer is displayed corresponds to the integer value in the slice dialogue, and by scrolling this value up and down the user can view each layer of the model. The user can also change which plane the slice value corresponds to, allowing slices to be viewed back to front, bottom to top and left to right, as shown in Figure 6.5. The slice dialogue affects all eight of the viewing options, i.e. each of the conductivities, driving forces and potential distributions. This allows for convenient viewing of any model parameter as well as the solutions in three dimensions.



Figure 6.4. Various aspects of the SPiso3D user interface. Directly below the menu bar (a) is a toolbar containing buttons that correspond to some of the more commonly accessed menu commands. These are, from left to right: create a new model; load and save a model; zoom window and zoom all; view the hydraulic (K), electric (R) and cross-coupling (X) conductivity distributions; view the hydraulic (H) and electric (E) driving forces; view the hydraulic pressure distribution in head (m) and pressure (Pa) units; view the electric potential distribution (V); edit the conductivity values; edit the driving forces; and finally the slice dialog (b) that allows viewing of different layers of a 3D model. Along the bottom of the window is a status bar (c) where information and notifications appear and coordinate data is displayed to the right (d).



Figure 6.5. Viewing the calculated hydraulic potential distribution for a simple $3 \times 3 \times 3$ element model. Shown are four slices in the *z* direction (*xy* plane, left), four slices in the *x* direction (*yz* plane, centre) and four slices in the *y* direction (*zx* plane, right). If *n* is the number of elements in a direction, there will be n + 1 slices to view in that direction because each element has two faces in each direction. Viewing separate slices in different planes is achieved with the slice dialogue in the toolbar. This feature can be used to view the distribution of any parameter in a 3D model.

Along the bottom of the window is a status bar where a variety of information can appear. This includes a brief description of each menu item when moused over, instructions once a command is selected, the time and number of iterations (should an iterative solution be selected) required for the calculation of a solution, notification of the successful completion of commands as well as possible errors. The end of the status bar contains coordinate information. Clicking the mouse in the window will yield the x and y coordinates, while pressing the button and sliding the mouse before releasing yields the coordinates at the release point and the x and y distances from the press point in brackets.

6.3. Modelling a streaming potential problem

In order to model a streaming potential problem the program requires a defined geometric region and the hydraulic, electric and cross-coupling conditions affecting that region. The logical starting point is to define the region by setting its coordinate boundaries and dividing it into elements. This is achieved in SPiso3D through the *New model* and *Coordinates* commands in the *Model* submenu.

The *New model* command opens a small dialogue window, shown in Figure 6.6, where the user selects the number of elements in each dimension to divide the region into and the type of element to use. Included in this window are two fields that display the calculated total number of elements and nodes for the selected values. The total number of nodes will also be the number of equations that will have to be simultaneously solved for the model.

The *Coordinates* command opens a larger dialogue window, shown in Figure 6.7, where the user can input the x, y and z coordinates for each of the eight corners of the region. The program will divide the region evenly into the selected number of elements and display the spacing for each dimension in three fields at the bottom of the window. The user has the option of modifying these default spacings so that areas of greater interest can contain more elements.

Although the original defined region is limited to a hexahedral shape, the locations of individual nodes can be moved with the *Move nodes* command in the *Model* submenu. This essentially allows any unusually shaped region to be modelled as well as modification of the element shapes within the region. When the command is chosen the user can simply select nodes and move them to new positions using the mouse. For 3D models the option is available to move all corresponding nodes in each slice to the new location simultaneously. The new coordinates appear in the right-hand section of the status bar.

Once the region's geometry and element discretisation have been defined, the hydraulic, electric and cross-coupling conditions can be input. This is achieved by assigning specific hydraulic and electric boundary conditions to individual nodes and by assigning different hydraulic, electric and cross-coupling conductivity values to individual elements. The *Conductivities* command is used to assign conductivity values to elements, and the *Driving forces* command is for assigning boundary conditions to nodes, both found in the *Model* submenu.



Figure 6.6. The *New model* command in the *Model* submenu lets the user define the number of rows, columns and slices the model is divided into. The type of element used can also be selected: 2-D and 3-D versions of linear and parabolic elements are available. The total number of elements and nodes are calculated and displayed.



Figure 6.7. The *Coordinates* command allows the user to define the model geometry through the eight corner coordinates and the element spacing in each direction.

The *Conductivities* dialogue window, shown in Figure 6.8, contains three tabs that can be selected, one each for the hydraulic, electric and cross-coupling conductivity distributions respectively. Each tab contains ten colour-coded conductivity combinations that each consist of an x, y and z conductivity value, allowing anisotropic conditions. The user selects one of the ten combinations, modifies its values if desired, and then assigns the combination to individual elements by selecting them in the main window. Elements are selected by clicking on them with the mouse and several can be selected simultaneously by clicking and dragging a rectangle. The selected element will immediately change colour to match that of the chosen conductivity combination.

Clicking on a different tab will change the display in the main window so that it shows the corresponding conductivity distribution. Buttons at the bottom of each tab allow the user to make the current combination isotropic, make all ten combinations isotropic, or to copy the conductivity distribution to the next tab. The checkbox at the very bottom gives the user the option to simultaneously assign the conductivities to all corresponding elements in each slice for 3D models.

The *Driving forces* dialogue window, shown in Figure 6.9, works in the same manner as the *Conductivities* window. There are two tabs, one for hydraulic driving forces and one for electric driving forces. There are ten possible combinations for each; however instead of anisotropic values, the user can assign either an essential or natural boundary condition, or both. Boundary conditions were discussed in the previous chapter. For hydraulic conditions, the pressure can be assigned as either a head value or as a point source. For electric conditions, a potential value can be prescribed or a point current source assigned.

In the main window, the model is drawn as a wire-frame mesh and the nodes appear as coloured spots. The driving forces are assigned to individual nodes by selecting them in the same manner as elements were selected for assigning conductivities. The selected nodes will change colour to match that of the chosen driving force. The option to assign the values to all corresponding nodes in other layers for 3D models is again available as a checkbox under the tabs.

As mentioned in the previous chapter, the first step in numerically modelling the streaming potential phenomenon is to obtain the hydraulic potential distribution. This can be achieved once the hydraulic conductivity distribution has been defined and the hydraulic driving forces have been applied. It should be noted that the program will not proceed unless a hydraulic driving force has been assigned to at least one node. Without this, the force vector of equation 5.19 remains equal to zero and the system of equations becomes ill-conditioned. Of course without a driving force, water will not flow and thus no streaming potentials would occur. For the case of an earth dam, the water levels on each side of the dam can be assigned to the nodes on the faces of the dam.

The hydraulic potential distribution is obtained using the *Solve hydraulic* command in the *Model* submenu. If no hydraulic driving force has been assigned, the user is notified through a status bar message. The program solves the hydraulic system of equations using a direct sparse solving subroutine. The term sparse can be used to

🐓 SPiso3D - zeep2	Conductivities				
File Display View Model Help	Hydraulic Electrical Cross-coupling				
	Bulk Electric Conductivity [S/m]				
	Kx Ky Kz				
	C 0.01 0.01 0.01				
	0.005 0.005 0.005				
	0.001 0.001 0.001				
	0.0005 0.0005 0.0005				
	0.0001 0.0001 0.0001				
	0.00005 0.00005 0.00005				
	0.00001 0.00001 0.00001				
LAHAHAU	0.000005 0.000005 0.000005				
R R R R	0.000001 0.000001 0.000001				
	C 1.0000000E-30 1.0000000E-30 1.0000000E-30				
	Ki x = Ki y = Ki z All Kx = Ky = Kz Copy to Next ->				
Match conductivity values to elements	Select all elements in the Z-direction OK				

Figure 6.8. The *Conductivities* command allows the user to modify values for the hydraulic, electric and cross-coupling conductivities and subsequently assign these values to individual elements. Values can be made anisotropic in three directions. Here electric conductivities are being assigned to elements in the second slice; however the checkbox at the bottom of the conductivities window ensures that the corresponding elements in the remaining slices are assigned the same values.



Figure 6.9. The *Driving forces* command is used to apply hydraulic and electrical boundary conditions to the model. Both essential and natural types of boundary condition can be applied. The forces are assigned to the element nodes by selecting them. Here a hydraulic pressure head is being assigned to nodes in the fourth slice.

define a matrix when most of its elements are zero. The generated stiffness matrix can be very large for a model of many nodes; however all of the non-zero terms will be concentrated near the diagonal. A sparse solver takes advantage of the sparseness of the matrix, saving on both time and computer storage. A direct solver factors the matrix into the product of two triangular matrices and then performs forward and backward triangular solve.

The time required to calculate a solution depends on the number of equations in the system and the processor and memory of the computer. Once completed, the hydraulic distribution is displayed in the main window and the results are written to an output file. The status bar shows the time required for the solution and the name of the file where the results were written.

When the *Solve hydraulic* command is selected a popup window appears asking whether the user would like the program to solve for the phreatic surface in the region. Selecting yes begins an iterative process where the program attempts to localize the water table throughout the model. For each iteration, the hydraulic potential distribution is recalculated while the nodes nearest to the free surface are relocated and the elements above the free surface are made impervious, preventing the upward flow of water. During the iterative process, the free surface variations are displayed in the main window. The process can be quite memory demanding for 3D models with many elements as several solution vectors are stored simultaneously during the process.

Once the hydraulic potential distribution has been calculated and the hydraulic and cross-coupling conductivities have been assigned, the electric streaming potential distribution can be solved for using the *Solve electric* command in the *Model* submenu. Unlike solving the hydraulic problem, no electric driving forces need necessarily be applied to any nodes by the user before solving the electrical problem. This is because the program first calculates the conduction current source terms from the hydraulic potential values and the cross-coupling conductivity distribution according to equation 5.25. These source terms are added to the force vector of equation 5.24, preventing the system of equations from being ill-conditioned.

The program solves the electrical system of equations using the same direct sparse solving subroutine used for solving the hydraulic problem. The results are displayed in the main window and added to the same file that contains the hydraulic potential distribution. The calculated conduction current source term values are also written to this output file.

Any aspect of the model can be modified at any point, even its geometry or number and type of elements. If the number or type of element is changed, however, then the conductivity values and any driving forces will have to be reassigned to the new elements and nodes. The hydraulic potential distribution can be calculated before any electrical conditions have been assigned. If changes are made to the geometry or the hydraulic conditions, then the hydraulic potential distribution should be recalculated before solving for the new electric potential distribution.

6.4 Modelling flow through an embankment dam

The program has been tested in an attempt to model real earth dam conditions using a couple of the investigated dams discussed in chapter 7. The purpose of the program was to provide insight into the streaming potential phenomenon, and perhaps it could help to explain SP measurements obtained in the field. Two dam sections were modelled, a hydro-electric dam with a moraine core and a mine tailings dam built as a composite of moraine and mine waste material.

6.4.1 The Ligga hydro-electric dam

The Ligga hydro electric dam contains a central sealing core of moraine that is supported by layers of gravel and rock fill. A three-dimensional model was created consisting of 30 columns, 13 rows and 6 slices, for a total of 2340 elements. Linear isoparametric elements were selected, bringing the total number of equations to 3038. The spacings in the *x* and *y* directions were modified together with the coordinates of individual nodes in an attempt to match the cross-section of the dam (see Figure 7.3). The spacing of the six slices in the *z* direction. The boundary at the other end of the model would act as impervious and would thus simulate a mirrored infinite length in the opposite direction.

The hydraulic conductivity distribution of the model is presented in Figure 6.10 together with the values used, which were partially taken from the section presented in Figure 7.3 and partially assumed. The hydraulic boundary conditions consisted of the water levels on the faces and are shown in Figure 6.11. The hydraulic potential distribution was solved for employing the option to iteratively locate the phreatic surface. The calculated pressure distribution and the free surface boundary are presented in Figure 6.12. It can be seen that the low downstream water level and high impermeability of the core material have produced a steep decline of the water table through the core.

The electric conductivity distribution was roughly approximated by interpreting resistivity data and by assuming resistivity values for the areas where no resistivity measurements were performed. A logical assumption would be low resistivity values below the water table and higher values near the surface. The assumed electric conductivity distribution is presented in Figure 6.13. A uniform cross-coupling conductivity distribution was assumed seeing as any assumptions concerning it would be pure speculation. No electric boundary conditions were assigned to any nodes, i.e. the electric potential distribution was solved for based solely on the calculated conduction current source terms.

The calculated streaming potential distribution is presented in Figure 6.14. The potential distribution on the downstream face does not appear to match the SP measurement values obtained in the field (see Figure 7.5); however very little resistivity data was collected on this dam and thus the assumed electric conductivity distribution could be inaccurate. On the other hand, the SP values obtained from most of the downstream face displayed a noticeable correlation to the topography as seen in Figure 6.15, i.e. a hillside streaming potential pattern. Removing this trend would result in the lower half of the face displaying a relatively flat negative SP variation
and increasingly positive values toward the crest, which better matches the model's potential distribution. Increasing the resistivity in the model toward the downstream crest would provide an even better match.

Because no variation of any condition was introduced in the *z* direction, all seven slices appeared the same. In order to demonstrate the program's capabilities in three dimensions, a seepage path was introduced through the dam as illustrated in Figure 6.16. Only the elements shown in the first slice were altered and assigned higher hydraulic conductivities and not the corresponding elements in the other slices.



Figure 6.10. Hydraulic conductivity values assigned to elements of the three dimensional SPiso3D model of the Ligga dam (see Figure 7.3). The model is shown in orthographic mode to reduce clutter.



Figure 6.11. Hydraulic driving forces for the Ligga dam model consisting of the water levels on the upstream and downstream faces. The values are assigned to the nodes located on each face that are at or below the water level. Note that areas where the nodes appear crowded can simply be zoomed.

Because the hydraulic conditions were altered, the pressure distribution had to be solved for again together with the free surface. The resulting pressure distribution in the slice containing the seepage path is presented in Figure 6.17. As was expected, the water table rose in the dam near the seepage path. The remaining slices showed that the further the slice was from the seepage path, the more its pressure distribution resembled the original shown in Figure 6.12.



Figure 6.12. Representation of the calculated phreatic surface and the hydraulic potential distribution throughout the dam. The obtained pressure values are also written in table form to an output file for closer examination.



Figure 6.13. The assumed electric conductivity distribution used for the Ligga dam model. Simply taking the reciprocal of the values yields the electric resistivities.



Figure 6.14. The electric potential distribution calculated by SPiso3D for the Ligga dam model. The values are also written to the output file containing the hydraulic potential distribution.



Figure 6.15. Plot of SP measurement values obtained from the Ligga dam versus elevation. The correlation is indicated by the best fit line. Removing this trend results in positive values on the upper slope and a relatively flat negative distribution below the 85 metre point.

The original electric conductivity distribution was altered to reflect the increased water level in the vicinity of the seepage path. The resistivity distribution for the slice containing the seepage path is displayed in Figure 6.18. No changes were made to the cross-coupling conductivity value. The calculated electric potential distribution for the slice containing the seepage path is presented in Figure 6.19. It can be seen that the original potential distribution had altered due to the seepage path. The potential values on the upper half of the downstream face had decreased. Just as with the hydraulic potential distribution, the electric potential distribution became closer to the original distribution with each subsequent slice further from the seepage path.



Figure 6.16. Introduction of a seepage path through the core material in the Ligga dam model. The elements forming the path, located in the first slice of the 3D model, are assigned lower hydraulic conductivities.



Figure 6.17. The calculated hydraulic potential distribution in the vicinity of the seepage path. Notice the elevated water level through the core and downstream half. Scrolling through the remaining slices of the model, achieved by the slice window in the toolbar, shows a return to the original distribution with distance from the seepage path.



Figure 6.18. Modifications made to the original electric conductivity distribution in the vicinity of the seepage path to account for the raised water level.



Figure 6.19. The electric potential distribution calculated by SPiso3D for the first slice of the Ligga dam model containing the seepage path. Scrolling through the remaining slices of the model again shows a return to the original distribution, shown in Figure 6.14, with distance from the seepage path.

6.4.2 The Aitik mine tailings dam

Unlike the previous dam, the Aitik mine tailings dam was built without a central sealing core of moraine. Instead, the dam was built up simply as a composite mass of moraine and waste rock. A two-dimensional model was created consisting of 50 columns and 32 rows, for a total of 1600 elements. Linear elements were again used, bringing the total number of equations to 1683. Notice that a two-dimensional model generates far less equations to be solved than a 3D model does, even when a relatively dense grid is used. Two dimensional models can be used when it is not necessary to model any variations in the z direction. Two dimensional elements are treated such

that they are assumed to extend infinitely in the third dimension. The geometry of the model was interpolated from geodetic measurements taken on the dam. The default even spacings in the x and y direction were not altered. Only the shapes of some elements along the downstream face were altered in order to reflect the dam topography.

Because the dam was built as a solid mass, a uniform hydraulic conductivity distribution was assumed for simplicity. The hydraulic boundary conditions again consisted of the water levels on the faces. Solving for the hydraulic potential distribution together with the location of the phreatic surface yielded a gently sloping water table through the body of the dam, presented in Figure 6.20.

Unlike the previous dam, a thorough electric resistivity investigation was performed together with the SP measurements. A number of resistivity pseudosections were taken moving up the downstream face of the dam (see Figures 7.10 and 7.12), providing excellent information regarding the subsurface electric conductivity distribution. All of the obtained pseudosections are presented in the next chapter; however good use was made of them here to model the electric conductivity distribution, shown in Figure 6.21. There were some difficulties with the pseudosections concerning the location of the water table in the lower portion of the dam; however the resistivity distribution was simulated in the model according to the pseudosections regardless.

The calculated streaming potential distribution is presented in Figure 6.22. An almost uniform negative potential covered most of dam, with the exception of a positive anomaly along the lower half of the downstream slope. This result actually compared quite well with the SP values obtained in the field (see Figures 7.11 and 7.13). This clearly illustrates the importance of complimenting SP data with resistivity measurements.



Figure 6.20. Two dimensional model of the Aitik mine tailings dam showing the phreatic surface and hydraulic potential distribution calculated by SPiso3D. A uniform hydraulic conductivity distribution produced a smooth water table through the dam.



Figure 6.21. The electric conductivity distribution used for the Aitik mine tailings dam model. The values and distribution were based on information gathered from resistivity pseudosections performed on the dam. Models can also be viewed as stretched to fill the window to more easily select individual elements or nodes.



Figure 6.22. The electric potential distribution calculated by SPiso3D for the Aitik mine tailings dam model. The distribution was found to resemble that measured in the field, indicating the importance of resistivity data for proper modelling and the interpretation of SP data.

7. Field measurements

During the course of this project a number of SP investigations were performed on a variety of embankment dams in order to provide a better understanding of the streaming potential phenomenon. Three hydro-electric dams of varying size were studied together with the selected tailing dams of three different mines. The hydro-electric dams are all located on the Luleå River in northern Sweden and include the Ligga, Messaure and Boden dams. The tailing dams belong to the Aitik, Kristineberg and Kiruna mines, which are also located in northern Sweden. The locations of the investigated dam sites are shown in Figure 7.1.

Both the hydro-electric and mining industries build and make use of impoundment dams and thus have a large interest in dam monitoring methods for safety reasons. Current dam monitoring methods typically include regular observation of water levels in standpipes and topographic surveys to monitor erosion and dam movements. The former method is limited by the number and location of standpipes and the latter method may not provide significant data before an imminent breach.

As previously mentioned, the SP method has been used in the past to identify potential seepage areas in a number of dam investigations. Despite good results however, the method is still not commonly used. The most likely reason for this is the difficulty in interpreting SP measurements due to a lack of understanding of the possible sources for SP anomalies. With a proper understanding of the streaming potential phenomenon and the other possible sources of self-potential anomalies, the SP method becomes a powerful tool for monitoring earth dams.

7.1 Field procedures

The sloping surface of an embankment dam can be steep and treacherous, often being covered by a protective layer of large loose rock that serves as both support and protection from erosion. Fortunately, as mentioned in chapter 2, it is relatively simple to perform SP measurements, particularly when using the absolute potential method which requires only one roving electrode. Resistivity measurements, on the other hand, complicate things by requiring additional current cables and electrodes, and obtaining a resistivity pseudosection can be quite awkward in steep, rocky areas.

7.1.1 SP method

To best facilitate movement on the dam surface, the absolute potential method was used on all dams. The reference electrode was placed outside of the study area, a distance from the base of the dam, and left stationary while the other electrode was moved along the surface. The instrument, a high impedance analogue voltmeter, was positioned at a convenient point roughly halfway up the dam face and connected to the electrodes by the cable reels. For each investigation the roving electrode was moved between relatively evenly-spaced points covering the study area of the dam surface. For most of the investigations, the exact location of each measurement point was recorded using a Leica Wild T1000 geodetic measuring instrument, also known



Figure 7.1. Map of northern Sweden showing the locations of the investigated dam sites. The Boden, Ligga and Messaure sites are hydro-electric dams built on the Luleå River while the Kristineberg, Aitik and Kiruna sites are mine tailing dams.

as a Total Station. This method provided the topography of the dam and the SP distribution in the form of the potential difference between the measurement points and the stationary base electrode. The results are presented here as SP contour maps shown with topography. The maps were created using either kriging or linear triangulation to interpolate the contours between measurement points.

Special hand-made Cu-CuSO₄ non-polarizing electrodes with wooden plugs were used for the SP measurements. Studies have shown this type of electrode to be very stable (Corwin and Butler, 1989; Wilt and Corwin, 1989). The electrodes consisted of plastic tubes filled with copper-sulphate solution constrained by the wooden plug at one end and a rubber stopper at the other. The connecting copper wire ran through the rubber stopper and formed a coil in the copper-sulphate solution. At each measurement point an effort was made to place the wooden plug in contact with relatively moist soil, when possible it was partially buried using a hand trowel. Each measurement value was required to stabilise before it was recorded. Unusual values were always confirmed by taking several nearby measurements.

The electrodes were calibrated for drift at the start and end of each measurement day by observing the potential difference between them when placed together in either a copper-sulphate solution or at a common point on the ground. Magnetotelluric effects were accounted for by making regular measurements of the SP difference between the stationary base electrode and a reference point conveniently selected somewhere near the measuring instrument. The variations in this reference value over time were recorded and the remaining measurements were linearly adjusted to remove the variations.

7.1.2 ER method

Unfortunately very little resistivity data was acquired in the early part of this thesis work concerning the three hydro-electric dams. This was due partly to equipment constraints and partly to the steep, rocky nature of the two larger dams. Later the software required for measuring and interpreting resistivity pseudosections was acquired and the method was included in the investigations of two of the three mine tailing sites.

The resistivity pseudosections were obtained using the pole-dipole configuration discussed in chapter 3 and a multi-channel system. This system consisted of a multi-channel board for connecting two current and eight potential electrodes to a voltmeter/ portable power source, in this case an ABEM SAS 4000 Terrameter with multi-channel software. A schematic representation of the system is presented in Figure 7.2. A common electrode spacing of two metres was used for all pseudosections. The potential pairs were located successively 1, 2, 4, 6 and 10 spacings (2, 4, 8, 12 and 20 metres) from the current pole.

All measurements were stacked at least four times, i.e. the instrument repeated each measurement four times and the standard deviation was recorded. Measurements containing significant deviations were repeated until an acceptable deviation was obtained. When the instrument finished recording the values corresponding to the five separate depths, the entire array was moved forward one spacing, unless the terrain difficulties dictated a larger displacement.



Figure 7.2. Schematic representation of the multi-channel pole-dipole system used for measuring resistivity pseudosections. C1 and C2 are the current electrodes and P1-P7 are the potential electrodes. Five measurements were taken corresponding to different depths by measuring the potential differences between P1 & P2, P2 & P3, P4 & P5, and P6 & P7. For the fifth measurement P6 & P7 were moved further from the current pole. Each potential electrode pair had a spacing of 2m.

Sections were taken both up the downstream face of the dam and along its length wherever reasonably suitable terrain existed. The locations of the resistivity measurement points were also recorded using the Total Station, allowing the effects of the topography to be accounted for by the interpolating program.

The measured sections were compiled into spreadsheets and analysed using an inverse modelling program called RES2DINV. As mentioned in chapter 3, inversion involves dividing the section into small elements and calculating the resistivity each element would require to produce the measured section. The resulting resistivity pseudo-sections are presented below.

7.1.3 Fixed electrodes

In addition to the roving electrode SP surveys and resistivity pseudosections, a number of fixed electrodes have been installed in two of the studied tailing dams. Performing the above mentioned surveys, although not very demanding, is time consuming. For regular dam monitoring a built-in electrode system is far more convenient. The electrodes could all be connected to a simple workstation that would connect and disconnect various electrode combinations, providing continuous monitoring of the SP and resistivity distributions in the dam. Such a system has already been installed on the Suorva Dam in northern Sweden and has been running for several years (Triumf et al, 1995). The results are interesting, to say the least; although problems were encountered with resistivity measurements when the ground was frozen.

The fixed electrodes used for this project consisted of small cloth bags filled with a mixture of soil, bentonite clay and copper-sulphate powder. The current electrodes for resistivity measurement had a similar construction, only without the copper-sulphate. The ends of the connecting copper wires were splayed out inside the bags in order to provide good contact with the mixtures. The electrodes were buried in pits dug along profile lines running up the face of the dam. The electrode pits were lined with a batting and filled with a finer soil in an attempt to keep the electrodes moist and stable. Cables were run from each electrode up to the crest of the dam to facilitate connections to the voltmeter/power source.

Although fixed electrodes were only installed on portions of the dam surfaces, the idea behind them is that they can be installed throughout the dam when it is built or renovated. This should not greatly increase construction costs and would permit monitoring of electric variations within the dam as well as on both the upstream and downstream surfaces.

7.2 Results from field investigations

The SP distribution maps and resistivity pseudosections resulting from the methods discussed above are now presented and discussed. Although measurements have been performed with the fixed electrodes at a few different time periods, the main purpose of the fixed electrodes is to monitor the SP variations over an extended period of time. At the time of the writing of this thesis, this portion of the project was still ongoing and interested readers are thus referred to the forthcoming report.

7.2.1 The Ligga hydro-electric dam

The embankment section of the Ligga hydro electric dam is approximately 40 metres high and between 200 and 300 metres long. It was first built in 1954 and expanded in 1982. The downstream face of the dam was covered by loose rubble ranging from large boulders to fine gravel as well as patches of scrub vegetation and small trees. The riverbed at the base of the dam consisted of bare rock with standing pools of water. An asphalted road ran along the top of the dam and the upstream face was similar to the downstream face, only with more soil and much denser vegetation. A schematic cross-section through the dam is presented in Figure 7.3.

The SP investigation covered a roughly 100 metre wide strip of the dam from the base of the downstream side to the water's edge on the upstream side. A single resistivity profile was taken along a step located midway up the downstream face (see Figure 7.5 for location). A Wenner configuration was used for this profile with an electrode spacing of five metres. The results are displayed in Figure 7.4. It is interesting to note how the SP values to a large degree reflected the resistivity profile. A local coordinate system was used and the locations of all measurement points were recorded using the Total Station.

The results of the SP investigation are shown in Figure 7.5. The overall pattern on most of the downstream face was that of a streaming potential on a hillside, with lower SP values at the top and higher values at the bottom. The slightly higher values on the upstream side were likely due to a higher moisture content of the soil and the denser vegetation on that side of the dam. Interestingly, at the time of the investigation a small leak was reported in the bottom left corner of the investigated area. This was confirmed by the relatively high SP anomaly observed along the lower left edge of the study area from roughly y = 0 to 40 metres, where trickling water was heard from between the rocks.

The larger negative SP anomaly in the centre of the dam crest could have been caused by something conductive in the ground, or it may have indicated the location of a seepage source near the crest. Two points with relatively high values were observed near the left edge of the crest and roughly 20 metres below the large negative anomaly. This type of small localised anomaly is most likely caused by local resistivity variations.



Figure 7.3. Schematic cross-section through the Ligga hydro-electric dam.



Figure 7.4. Resistivity profile along a step-road located midway up the downstream face of the Ligga dam, together with SP measurement values. The profile was obtained with a 5m Wenner electrode configuration.



Figure 7.5. SP distribution map compiled from total potential SP measurements performed on a section of the Ligga hydro-electric dam, shown with topography.

7.2.2 The Messaure hydro-electric dam

The Messaure hydro-electric dam was once northern Europe's largest dam. At 100 metres high, the Messaure dam is presently the second highest in Sweden, and it is over two kilometres long. This dam was built in 1963 and expanded in 1984. A recent renovation was performed in 2003, although the dam was not raised higher. A schematic cross-section through the dam is presented in figure 7.6.

The slopes of this dam were covered with loose rock averaging at roughly one metre in diameter, with a scattering of much larger boulders. Obtaining good contact with the ground through the blunt wooden plug of the electrodes was rather difficult on this dam. Slides of loose, sandy gravel extended from the crest, along which an asphalt road passed. There was no vegetation except along the crest and the steps on the downstream face, where some small thin trees had taken root. The investigation was made late in the fall and the ground became more and more frozen toward the bottom of the downstream face. Due to the rocky surface and frozen ground conditions, no resistivity measurements were attempted. No geodetic survey was performed either, thus the locations of the measurement points are only approximated. The investigation covered a 120 metre wide strip of the eastern part of the dam.

The results of the SP investigation are presented in figure 7.7. As in the Ligga Dam investigation, the Messaure dam displayed a characteristic hillside streaming potential distribution. The relatively low SP values at the bottom centre of the downstream face were likely due to the fact that those points were located beside a concrete well containing water and a lot of scrap metal. The small negative anomaly point in the centre of the slope at x = 95m, y = -7m represents where several old metal pipes protruded from the slope. The upper portion of the slope displayed vertical linear features where SP values were slightly higher than those to either side until they reach the road. These corresponded to large slides of sandy gravel, which would have contained more moisture through capillary action and thus produced stronger hillside SP patterns than the adjacent rock slopes. No seepage problems were expected on this dam and no inexplicable large anomalies were present in the data.



Figure 7.6. Schematic cross-section through the Messaure hydro-electric dam.



Figure 7.7. SP distribution map compiled from measurements performed on a section of the Messaure hydro-electric dam, shown with approximated topography.

7.2.3 The Boden hydro-electric dam

The hydro-electric dam investigated in the city of Boden was much smaller than the two previous dams. This dam was of particular interest because at the time it was reported to be experiencing leakage problems. The downstream surface was ideal for planting electrodes as it was uniformly covered by sandy soil and grass. The water level at the base of the downstream side was not far below the surface. Unfortunately the dam was so small that the study area had to be immediately adjacent to the large concrete and steel floodgates, which may have obscured SP anomalies. Also, traffic on the asphalt road traversing the dam was too frequent to allow any measurements to be taken on the upstream side. No geodetic or resistivity measurements were performed on this dam due to time constraints.

The results of this investigation, presented in Figure 7.8, were rather unusual. The typical hillside streaming potential pattern was not apparent, instead a large positive

SP anomaly appeared near the dam crest between x = 46m and x = 86m. Without knowledge of the resistivity distribution it is very difficult to explain this anomaly. It may have been an effect of the floodgates, located to the right of these measurements; however the anomaly did not appear to continue to the right. It may have simply represented the resistivity distribution in the dam, indicating a higher resistivity towards the crest. This could have been due to the high groundwater level at the base of the dam. The decreasing SP values to either side of the anomaly may have indicated more conductive areas with higher moisture content.

Three points existed along the crest where the SP values were relatively high compared to adjacent values, at coordinates x = 6 m, 26 m and 46 m. The size and locations of theses anomalies would indicate that they were not likely to represent streaming potential sources. The low SP anomaly in the upper left corner may have indicated a leakage source; however it was difficult to tell with the rest of the distribution appearing so unconventional.



Figure 7.8. SP distribution map compiled from measurements performed on a section of the Boden hydro-electric dam, shown with approximated topography.

7.2.4 The Aitik mine impoundment dams

The Aitik mine, owned by Boliden Mineral AB, is considered to be the largest open pit copper mine in Europe. Production commenced in 1968, and by 2012 it is expected that the northern side of the mine will reach a depth of 400 metres. Gold and silver are also produced here. Once the valuable ores have been extracted, the tailings are piped in slurry form to a large impounding reservoir, which is controlled by a very long earth dam. Water from this reservoir can be released through gates to a second smaller impounding reservoir, also restrained by a long earth dam. It should be noted that the Aitik earth dams were not built in the same manner as the above hydro-electric dams. Instead of using an impervious core with layers of supporting fill material, the dams were built as composite masses of local moraine, waste rock and slurry sand. In the upper reservoir, the suspended sediments in the slurry settle to form sandy layers on the bottom. In order to raise the dam higher, material is added to the crest and extended into the reservoir such that the layers of sediments form a portion of the new dam's foundation, as illustrated in Figure 7.9. This process is repeated whenever the dam needs to be raised.

The surface of the upper reservoir dam ranged from unsorted blast rock at the bottom to fine sulphide-enriched sand at the top and contained several step roads. The lower reservoir dam was primarily covered by medium-sized rocks and had a packed gravel road along the crest. Resistivity and SP measurements were performed on portions of both the upper and lower reservoir dams. Two 150 metre wide sections of the upper dam were investigated together with a 150 metre wide section of the lower dam. The general locations of the three study areas are shown in Figure 7.10. The locations of all measurement points were recorded using the Total Station and for this investigation site tied into the Swedish national coordinate system (RT-90).

Three resistivity pseudosections were obtained from each of the two study areas on the upper reservoir dam, two extending from the downstream toe to the upstream crest and the third taken along one of the steps running across the downstream face. Only one pseudosection was obtained from the lower reservoir dam due to the rocky nature of its surface. The section was taken along the upstream edge of the crest road. The electric pseudosections were numbered PS1 to PS7 for the order in which they were measured.

The locations of the three pseudosections obtained from the first study area are shown in Figure 7.11 together with the results of the SP investigation. The calculated resitivity pseudosections are presented in Figure 7.12. Sections PS1 and PS6 display relatively similar electric resistivity distributions. Both show high resistivity values (>1000 Ω m) from the second step downward and low values above this point, which decrease with depth. This difference in the resistivity pattern for the upper and lower portions of the dam was likely a reflection of the different construction materials. The



Figure 7.9. Schematic representation of the alternative technique used for the construction of the Aitik mine tailing dams. Layers of sediment accumulate on the bottom of the tailings pond and form a portion of the dam's foundation as it is raised. Although presented as distinct stages, the construction may also occur as a gradual, ongoing process.



Figure 7.10. Map of the Aitik mine area showing the locations of the three investigated areas. A and B represent the first and second study areas on the upper reservoir dam and C indicates the third study area on the lower dam.

enriched sand of the upper portion would have had a lower resistivity than the blast rock of the lower portion due to a higher moisture content held by capillary action.

Section PS5, taken along the lowest step road near the base of the dam, shows a relatively high resistivity that begins to decrease at a depth of roughly seven metres. There is some variation in this distribution along the profile, with some areas showing higher resistivity values than others. This is likely a reflection of different moisture conditions along the profile caused by the inhomogeneous construction of the dam. The low values along the bottom of the section indicate the influence of the water level in the dam; however these values are still higher than most of the distribution values above the second step.

The bottom ends of sections PS1 and PS6 also show decreasing resistivity values, indicating the water table as it approached the level of the lower reservoir. Besides at the base however, the water table did not appear as was expected in the lower portion of the dam. Resistivity measurements performed on water samples from the upper reservoir indicated very low resistivities (~5 Ω m), thus the water table should have appeared as a relatively low resistivity feature through the dam. It was suspected that the modelling program RES2DINV had problems interpreting the sharp resistivity gradient that was likely found in the lower portion of this dam.

The results of the SP investigation performed on the first study area, shown in Figure 7.11, displayed an SP distribution that reflected the information gathered from the resistivity pseudosections in several respects. First, there was very little variation in the SP values along the length of the dam, indicating that there were no local anomalies within this investigated section of the dam. Second, areas that displayed



Figure 7.11. SP distribution map compiled from measurements performed on the first study area of the upper reservoir dam at the Aitik mine site, shown with topography and the locations of resistivity pseudosections PS1, PS5 and PS6.

high SP values coincided with areas of high resistivity. This can be clearly seen along the length of section PS5. The low SP area on the face above the second step also coincided with the locations in the pseudosections where low resistivity areas appeared closest to the surface.

The SP values obtained along the water's edge were not particularly low however, given the water's very low resistivity. A possible explanation was that streaming potential sources were contributing to the SP anomaly above the second step. This is strengthened by the fact that this wide anomaly coincides with the positive anomaly in the centre of the first step.

A similar condition occurred at the water level of the lower reservoir, where slightly higher than expected SP values were observed. The low values on the face and the high values at the toe may have represented streaming potential sources, whether from hillside streaming potentials or a flow path through the dam. The steep gradient in SP values along the second step was probably a reflection of the different building materials in the dam, indicating a boundary where the hydraulic conductivity and possibly the cross-coupling conductivity changed.



Figure 7.12. Resistivity pseudosections PS1, PS6 and PS5 obtained from the first study area of the upper reservoir dam at the Aitik mine site.



Figure 7.13. SP distribution map compiled from measurements performed on the second study area of the upper reservoir dam at the Aitik mine site, shown with topography and the locations of resistivity pseudosections PS2, PS3 and PS4.

The locations of the three pseudosections obtained from the second study area are shown in Figure 7.13 together with the results of the SP investigation. The calculated resitivity pseudosections are presented in Figure 7.14. They display resistivity distributions very similar to those obtained from the first study area. Sections PS2 and PS4 show similar low resistivity values above the second step and high resistivities below it. Section PS 2, taken along the lowest step, displays a uniformly layered resistivity distribution that decreases with depth and has almost no lateral variation, unlike section PS5. Just as in the previous study area, the water table in the lower portion of the dam was not well represented in the pseudosections.

The results of the SP investigation performed on the second study area, shown in Figure 7.13, were also similar to those obtained from the first study area. However, although the electric resistivity measurements were very similar for the two areas, the SP distributions do differ to some degree. For example, section PS2 shows slightly lower resistivity values along the bottom of the profile than section PS5; however it also shows a more uniform, slightly higher resistivity to a depth of roughly six metres. The SP values obtained along the PS2 line were however considerably lower than those obtained along the PS5 line. This would indicate that streaming potentials were influencing the two sites differently.



Figure 7.14. Resistivity pseudosections PS2, PS3 and PS4 obtained from the second study area of the upper reservoir dam at the Aitik mine site.

Besides the lower SP values along the first step, the rest of the SP distribution was quite similar to that of the first study area. The SP values on the face above the second step were slightly higher than those observed at the first site, which could have been a further indication of different streaming potential influences on the two sites. The steep gradient in SP values along the second step appeared much as it had in the previous study area.

At the second investigated area it was possible to take SP readings on a sandbar that extended slightly into the upstream reservoir. These readings produced a slightly negative anomaly that may have indicated a streaming potential source where water seeped into the dam; however the differences from surrounding values were not very large.

Of possible interest was the positive anomaly in the centre of the second step, which was inline with a slightly negative anomaly on the slope above it as well as the negative anomaly on the upstream side. These may have been indications of a seepage path through this section of the dam. These anomalies shared similarities with those observed in the previous study area, although these were much smaller and the positive anomaly occurred on the second step rather than the first.



Figure 7.15. SP distribution map compiled from measurements performed on the investigated area of the lower reservoir dam at the Aitik mine site, shown with topography and the location of resistivity pseudosection PS7.



Figure 7.16. Pseudosection PS7 calculated from the resistivity measurements performed along the crest of the lower reservoir dam at the Aitik mine site.

The location of the final pseudosection obtained from the third study area on the lower reservoir dam is shown in Figure 7.15 together with the results of the SP investigation. The calculated resistivity pseudosection is presented in Figure 7.16. The resistivity values are seen to be high near the surface and begin to decrease with depth as they approach the water level in the dam. The variations along the length of the profile were likely the effects of inhomogeneities in the dam as well as the fact that good contact for the current electrode was sometimes difficult to achieve.

The overall SP pattern shown in Figure 7.15 is that of a hillside streaming potential with negative values at the top and positive values toward the base. An exception is the positive anomaly located on the crest of the dam approximately two thirds of the way along the PS7 profile line. This point coincided with the highest resistivity value obtained in pseudosection PS7, located on the surface at 113 metres along the profile. The value was however limited to a small point in the pseudosection, whereas the SP anomaly was considerably larger and thus may have represented a streaming potential source. Interestingly this section of the dam was reported as leaking at the time of this investigation.

7.2.5 The Kristineberg mine impoundment dam

The Kristineberg mine, also owned by Boliden Mineral AB, is Sweden's deepest mine at 1170 metres. The Kristineberg deposit, containing zinc, copper, lead, gold and silver, was discovered in 1918 and mining was begun in 1940. The ore is now transported 100km east to Boliden for processing; however before 1991, Kristineberg had its own refinery that processed ore from several mines that were once active in the surrounding area in addition to its own.

The Kristineberg mine tailings reservoir is located in a small valley and is controlled by an earth dam at one end. This dam is old and of the same construction type as discussed above for the Aitik earth dams. The dam has been raised a number of times using a range of different construction materials and techniques. The downstream face of the dam was soil and grass-covered and became more rocky toward the base. Signs of erosion were evident on the lower half where channels sometimes a metre wide and 0.5m deep had been washed out. The upstream face consisted primarily of large rocks and there was a gravel road running along the width of the crest. At the time of measurement the dam crest was being raised by a half metre. A 300 metre wide section of the dam was investigated, the location of which is shown in Figure 7.17. Six electric resistivity pseudosections were obtained, five extending from toe to crest and one running along the dam near the toe of the downstream face. The locations of the pseudosections are shown in Figure 7.18 together with the results of the SP investigation.

Fixed electrode arrays were also installed along the five vertical pseudosection lines in order to allow future monitoring of SP and resistivity variations. The locations of these are also indicated in Figure 7.18. Three potential and two current electrodes were installed along each pseudosection line. The locations of all measurement points were recorded using the Total Station and tied into the Kristineberg mine local coordinate system.

The six resistivity pseudosections obtained from the Kristineberg tailings dam are presented in Figures 7.19 and 7.20. Pseudosections PS1 and PS3 to PS5 all display similar resistivity distributions up the face, with high resistivity values at the surface that decrease with depth. Section PS2 shows an abnormally high resistivity structure below the surface. Although the individual measurement values from this section appeared reasonable, the resulting pseudosection is questionable. This was supported by the unusually high error obtained for the inversion model.

Pseudosections PS1 to PS5 all show a low resistivity area extending from the bottom of the section to a point on the surface at the crest of the dam. Due to the irregular construction of the dam, this low resistivity area was likely a reflection of variations in the properties of the construction materials used.

Section PS6 was measured along the length of the study area near the base of the downstream side. The resulting pseudosection displays a relatively even distribution along its length with high resistivities at the surface that decrease with depth. The values appear to decrease sharply at a depth ranging from approximately two metres at the beginning of the profile to four metres at the end. The lateral variations that occur along the length of the profile were again most likely due to inhomogeneous conditions in the dam.

The results of the SP investigation performed on the Kristineberg tailings dam, shown in Figure 7.18, were somewhat unusual in that there was no sign of a hillside streaming potential pattern. In fact, high values predominated along the crest and low values covered the slope. The resistivity pseudosections supported the SP distribution along the crest to some degree; however there was not so much correlation along the slope. The negative anomalies occurring around mid-slope may have been caused by streaming potential sources, indicating seepage paths in the dam.

A positive SP anomaly was observed along the bottom near the left edge of the study area. This may have been due to streaming potential sources where water flowed from the dam. Smaller positive anomalies occurred further along the base of the dam. These were also likely caused by streaming potentials, particularly as visible signs of seepage were observed at these points.



Figure 7.17. Map of the Kristineberg mine area showing the impounding reservoirs and the location of the investigated area.



Figure 7.18. SP distribution map compiled from measurements performed on the Kristineberg mine tailings dam, shown with topography. Also represented are the locations of each resistivity pseudosection as well as the fixed electrode installations.



Figure 7.19. Resistivity pseudosections PS1, PS2 and PS3 obtained from the Kristineberg mine tailings dam.



Figure 7.20. Resistivity pseudosections PS4, PS5 and PS6 obtained from the Kristineberg mine tailings dam.

7.2.6 The Kiruna mine impoundment dam

The iron ore mine in Sweden's northernmost city is operated by LKAB, a state-owned mining and mineral processing company. Open pit mining in Kiruna started in 1902 and today operations continue deep underground. Current plans for expanding the mine actually include relocating parts of the city.

The entire length of one of the Kiruna mine tailing dams was investigated using the total potential SP method. The location of the dam is indicated in Figure 7.21. Because of the length of the dam, nearly four kilometres, relatively wide spacings were used between measurement points. While interpreting the data, several areas of the dam displaying unusual SP variations were identified and further measurements were performed on these sections. The investigated dam was built using the construction technique common to hydro-electric dams; i.e. the dam possessed a vertical core consisting of local moraine supported by slopes of waste rock.

Shortly after the preliminary SP investigation was performed, LKAB began work to raise the dam by 1.3 metres. The opportunity was taken under this renovation to install fixed electrodes in the sections of the dam where unusual SP variations were observed for future monitoring of SP and resistivity variations. Four lines of fixed electrodes were installed across the dam, each consisting of one current electrode and four potential electrodes. The four potential electrodes were installed at the toe of the downstream slope, midway up the downstream slope, and one each at the downstream and upstream crests respectively. The current electrode was placed a couple of metres from the downstream crest toward the centre of the dam.



Figure 7.21. Map of the Kiruna mine area showing the impounding reservoirs and the location of the investigated dam.

At the time of the original SP investigation no resistivity measurements had been performed on the Kiruna mine dam; however several ground-penetrating radar profiles were obtained. The investigation itself was performed by GeoVista AB, a Swedish geophysics consulting company that cooperated with the Luleå University of Technology for this project. At the time of writing this thesis the SP investigation had been very recently repeated in order to observe the SP variation with time over the entire dam. Readers interested in the results of these investigations are referred to the forthcoming reports.

8. Conclusions

The main results of the work performed for this thesis are now summarised and discussed. Also presented in this final chapter are the ideas and suggestions for future research that arose from the work performed here.

8.1 The main results

A finite element forward modelling program was created to obtain the secondary potential distribution throughout a three-dimensional domain caused by a primary flow. The thermodynamic theory of coupled flows was used to relate the secondary potential to the primary flow. The program can be used to model the streaming potential phenomenon, where the primary flow is the hydraulic pressure gradient throughout the domain and the secondary potential is the electric current produced. The hydraulic pressure distribution in the domain is first calculated from driving forces such as pressure differences or point sources or sinks. The pressure distribution is then used to calculate the conduction current sources that are generated by the fluid flow. The resulting electric potential distribution is then calculated from the conduction current sources.

In order to model streaming potentials, the program requires four parameters. An initial hydraulic driving force and the hydraulic conductivity distribution in the domain are required to solve for the hydraulic pressure distribution. Next, the cross-coupling conductivity distribution in the domain is necessary to calculate conduction current source terms. Finally, the electric conductivity (or resistivity) distribution in the domain is required to obtain the resulting electric potential distribution. An optional input parameter is the ability to assign external current sources in addition to the calculated conduction current sources, effectively allowing resistivity measurements to be simulated.

The model domain can take any three-dimensional shape and can be divided into elements as desired. The individual elements can be assigned separate hydraulic, electric and cross-coupling conductivity values, creating an inhomogeneous anisotropic domain with three separate conductivity distributions. Four different types of finite element are available to choose from; two- and three-dimensional versions of isoparametric elements with either linear or quadratic interpolating polynomials. The program has been made fully graphical, allowing the user quick and easy access to information at any particular point of the domain.

The hydraulic potential distributions obtained by the program match closely with analytical solutions. The program can also determine the location of the phreatic surface throughout the model domain while calculating the hydraulic potential distribution. The obtained electric potential distributions reflect the calculated conduction current sources as well as variations in the electric and cross-coupling conductivity distributions. The results from simple models containing point pressure sources and sinks matched well with those from the finite difference electric potential program SPPC as well as analytical solutions. Models simulating real earth dam conditions produced potential distributions that showed reasonable agreement with field measurements.

Investigations were performed on a number of embankment dams using the selfpotential method. Electric resistivity measurements were performed on several of the studied dams to compliment the SP data. The results provided a good representation of typical SP distributions that can be expected on earth dams built by both the mining and the hydro-electric industries with proper equipment and field methods. The locations of possible seepage zones were identified in several of the investigated embankment dams.

The resistivity pseudosections in particular provided valuable information for the interpretation of SP data. The pseudosections also took much of the guesswork out of detailing the electric conductivity distribution when attempting to model the embankment dams with the computer program developed in this thesis. The results of a model with a proper resistivity distribution provided an SP distribution very similar to that obtained in the field.

8.2 Prospects for future research

In the process of doing research work there are always interesting avenues that cannot be explored due to financial or time constraints. The work started for this project concerning the fixed electrode installations in the mine tailing dams was ongoing at the time of writing this thesis. Additionally, soil and water samples were being collected and analysed. The collective data should provide a better understanding of the differences that can be expected from SP and resistivity measurements taken on hydro-electric and mine tailing dams.

The modelling program created for this thesis functioned well; however it required knowledge of the separate conductivity distributions within the model domain. The natural progression from this forward modelling technique would be to introduce inversion modelling, where the program would attempt to iteratively solve for the conductivity distributions based on input SP values.

A better knowledge of practical streaming potential coefficient values would also be of great benefit for estimating cross-coupling conductivities. As mentioned, there is very little documentation concerning this attribute, particularly for real-world soil and rock conditions. A greater knowledge base should be built up with both field studies and laboratory experiments.

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